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OFFICE OF THE REGISTRAR ... 1950

The MATHEMATICS STUDENT

A Quarterly Dedicated to the Service of Students and Teachers of Mathematics in India

Vol. XVII SEPTEMBER—DECEMBER 1949 Nos. 3 & 4

Editor
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Andhra University, Waltair, South India

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Hon. Secretary, Indian Mathematical Society, Bhaskarapuram, Mylapore, Madras
1950

THE INTERNATIONAL CONGRESS OF MATHEMATICIANS

CAMBRIDGE, MASSACHUSETTS, U.S.A. AUGUST 30 - SEPTEMBER 6, 1950

An International Congress of Mathematicians will be held in Cambridge, Massachusetts, in 1950 under the auspices of the American Mathematical Society. The Society had hoped to be host to a Congress in September, 1940. However, the outbreak of World War II made it necessary to postpone the Congress and, consequently, there has been no international gathering of mathematicians since 1936. It is the sincere hope of the American Mathematical Society that the gathering in 1950 will be a truly international one, with all countries well represented. The Council of the American Mathematical Society has voted unanimously to hold a Congress which will be open to mathematicians of all national and geographical groups.

Former Congresses

In connection with the World's Columbian Exposition in Chicago in 1893 there was an International Congress of Mathematicians. The first Congress in Europe was held at Zurich in 1897. Except for omissions necessitated by the two World Wars, there have been similar gatherings about once every four years. These have all been in Europe except that of 1924, which was at Toronto. The most recent Congress was held at Oslo in 1936.

At the most recent Congresses, the number of countries represented has been about 40, the number of participants about 600, and the number of short papers presented about 250.

Time and Place

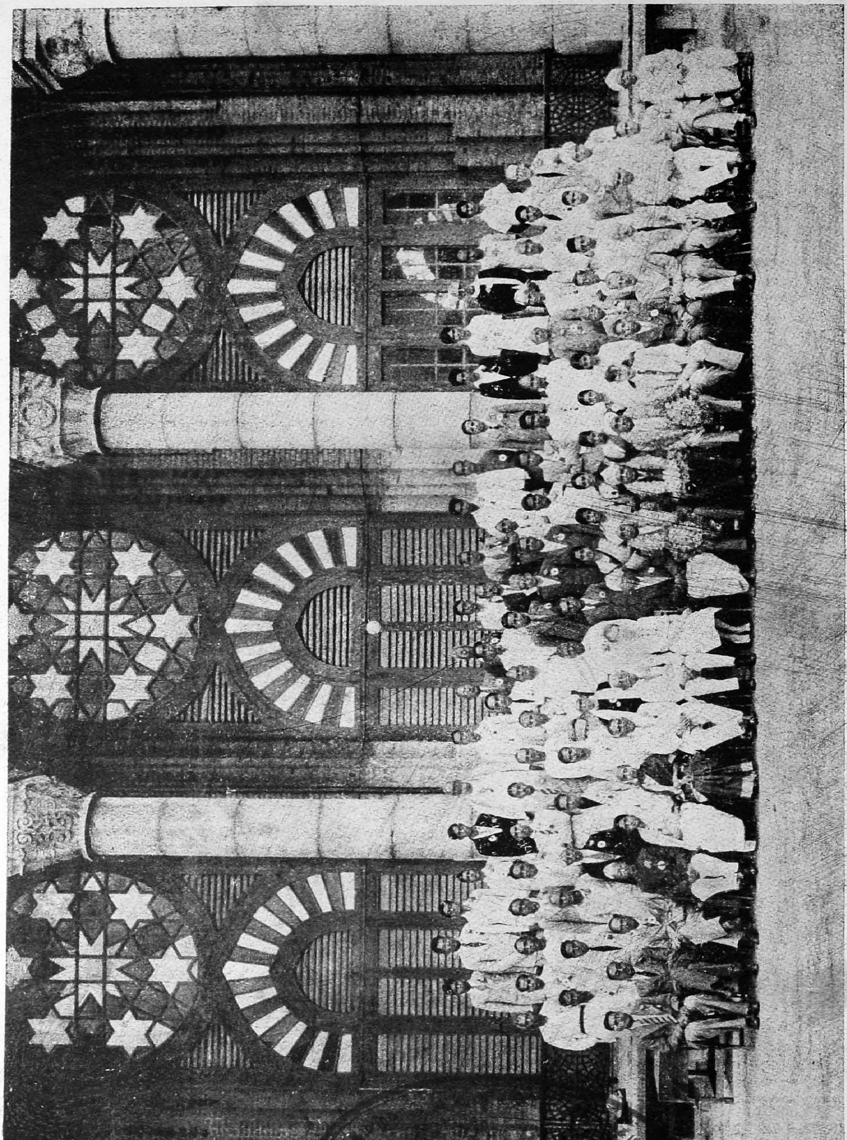
The dates of the Congress have been fixed as August 30-September 6, 1950. The American Mathematical Society, the Mathematical Association of America, and the Institute of Mathematical Statistics, which usually meet jointly in the summer, will omit their summer meetings in 1950 so that their members may more easily participate in the Congress. Harvard University will be the principal host institution. Other institutions in metropolitan Boston will also participate by arranging special entertainments. Mathematicians so desiring will be housed in the Harvard University dormitories and meals will be served in the University dining rooms. Mathematicians from outside the United States and Canada who become members of the Congress and who travel to the United States primarily for the purpose of the Congress may reasonably expect to receive room and board at Harvard University without charge during the period of the Congress. There will be accommodations for members of families at a modest charge, special provision being made for the care of children. Those who prefer to live in hotels can be comfortably provided for in Cambridge or Boston at their own expense.

Organization

The Organizing Committee has invited more than a score of outstanding mathematicians to deliver stated addresses. There will be Conferences in four different fields; these are more fully described below. There will be seven Sections for the presentation of short contributed papers and by invitation of the Section Chairmen there will be a small number of half-hour addresses as part of the Section programs.

In recent years mathematicians have been much impressed by the success of the conference method for presenting research in fields in which vigorous advances have just been made or are in progress. There will accordingly be a coordinated program of formal lectures and informal open discussion, and the stated addresses will be integrated as far as possible with the work of the Conferences. The following list of topics gives an indication of the nature of each of the Conferences:

(Continued on cover page 3)



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SIXTEENTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

MADRAS 26th to 29th DECEMBER 1949.

Sitting :— D. V. Rajalakshman, P. B. Patnaik, Hansraj Gupta, C. N. Srinivasiengar, M. Srirangamma, S. Pankajam, K. R. Gunjikar, D. D. Kapadia, Mrs. Tietjens, S. Mahadevan, A. Narasinga Rao, Sir. A. L. Mudaliar, Sir. C. V. Raman, M. H. Stone (*Chicago*), Ram Behari, H. Wold (*Upsala*), O. G. Tietjens, T. Vijayaraghavan, B. S. Madhava Rao, G. A. Srinivasan.

Standing 1st Row :— K. Rangaswami, B. C. Das, T. V. Ramamurti, T. Totadri Iyengar, V. K. Doraiswamy, S. Suryanarayanan, R. Krishnamurti, C. V. Subbarama Iyer, N. Vencoba Rao, Miss Madhava Rao, N. Kamalamma, S. N. Manikarnikamma, K. Padmavati, S. Indira, S. Saraswati, Balasinga Satya Nadar, S. Minakshisundaram, M. L. Chandratreya, M. M. Garde, A. A. K. Iyengar.

2nd Row :— V. Ramaswamy, V. Ganapati Iyer, T. K. Srinivasan, S. Raghavachari, A. A. Gnanadoss, C. S. Venkataraman, M. V. Subba Rao, T. S. Venkataraman, A. Sankaranarayanan, K. L. Varma, P. L. Bhatnagar' V. R. Tiruvenkatachar, G. V. Bhagavat, N. Rama Rao, H. G. S. Sharma, A. S. Krishnamurti, S. Ganapathy' M. Venkatarama Iyer, K. V. Iyengar, M. Lakshmanamurti, P. V. Abdur Rahman, W. F. Kibble.

3rd Row :— D. W. Kerkar, R. Venkataraman, T. P. Chatterjee, M. Venkataraman.

4th Row :— K. Sundaresan, C. B. Madhava Rao, V. K. Balachandran, D. R. Kaprekar, M. S. Srinivasan, R. Venkatachalier, V. Ranganathan, S. M. Shah, S. S. Pillai, K. Venkatachaliengar, T. Venkatarayudu, C. R. Rao, D. B. Lahiri, G. L. Chandretraya, R. Ramanujam, Volunteer, S. Parameswara Iyer, Volunteer.

THE MATHEMATICS STUDENT

Vol. XVII] SEPTEMBER–DECEMBER 1949 [Nos. 3 & 4

SIXTEENTH CONFERENCE
OF THE
INDIAN MATHEMATICAL SOCIETY
MADRAS

26th, 27th, 28th & 29th December 1949

RECEPTION COMMITTEE

Patron:

His Excellency The Maharajah of Bhavanagar, Governor of Madras

Chairman:

Sir A.L. Mudaliar, Kt., B.A., M.D., F.R.C.O.G., Vice-Chancellor, Madras University.

Vice-Patrons:

Sir C. P. Ramaswamy Aiyar, Kt.
Hon'ble Sri K. Madhava Menon, B.A., B.L.
Sir S. V. Ramamurthy, Kt., I.C.S.
Sir Vepa Ramesam, Kt.
Sir Raja Muthiah Chettiar of Chettinad, Kt.
Sir Rm. Alagappa Chettiar, Kt., M.A., D.Litt.
Sri. K. Srinivasan, B.A.
Sri. T. S. Narayanaswamy

Vice-Chairman:

Dr. R. Vaidyanathaswamy, M.A., Ph.D., D.Sc.

CONFERENCE INAUGURATED BY

Sir C. V. Raman, Kt., F.R.S., N.L.

EXHIBITION OPENED BY

Sir C. P. Ramaswamy Aiyar, K.C.S.I., K.C.I.E.

PRESIDENT

Dr. A. Narasinga Rao, M.A., D.Sc., Andhra University, Waltair

Hony. Secretary:

Sri. S. Mahadevan, M.A., L.T., Mylapore, Madras

Treasurer:

Sri. N. Sundaram Iyer, M.A., Vivekananda College, Madras

Local Secretaries:

G. A. Srinivasan, M.A., L.T., Presidency College, Madras
D. V. Rajalakshman, M.A., M.Sc., F.S.S., University of Madras, Madras

Place: SENATE HOUSE, CHEPAUK, MADRAS-5

Programme

Monday, 26th Dec. 1949

10–30 a.m. INAUGURAL MEETING:

Prayer.

Welcome Speech by Sir A. L. Mudaliar, Vice-Chancellor.

Inauguration of the Conference by Sir C. V. Raman.

Reading of Messages.

Secretary's Report.

Presidential Address by Dr. A. Narasinga Rao.

Opening of the Exhibition by Sir C. P. Ramaswamy Aiyar.

Vote of Thanks.

National Anthem.

2–30 p.m. Symposium on 'Recent developments in harmonic analysis'
President: Prof. M. H. Stone. *Other speakers*:
Dr. K. Chandrasekharan and Dr. S. Meenakshisundaram

5–00 p.m. Tea—by the Reception Committee.

6–30 ,, Lecture by Sir C. V. Raman on "Luminiscence of Solids" with demonstrations.

Tuesday, 27th Dec. 1949

8–30 a.m. Lecture by Sir C. V. Raman on Diffraction Problems.

11–00 ,, Reading of Papers.

2–30 p.m. Symposium on "Time series Analysis"
President: Prof. Herman Wold. *Other speakers*:
Dr. C. R. Rao and Dr. N. S. Sastry.

4–00 ,, Group Photo.

4–30 ,, Local Excursion: Visit to Adyar.
Tea and Variety Entertainment at Kalakshetra, Adayar.

Wednesday, 28th Dec. 1949

8–30 a.m. Reading of Papers.

11–00 ,, Symposium on 'Fluid Mechanics' with film show on vortex motion.
President: Prof. O. G. Teitjens. *Other speakers*:
Dr. Ghatage and Dr. P. V. Abdur Rahiman

3–00 p.m. Symposium on 'Teaching of Mathematics'
President: Prof. A. Narasinga Rao.

5–00 ,, Local Excursions:
1. Visit to Presidency College
2. Visit to 'The Hindu Office'

6–30 ,, Film Show on Atomic Energy.

Thursday, 29th Dec. 1949

Local Excursions:
1. Red Hills Lake and Kilpauk Water Works.
2. Madras Christian College, Tambaram
3. Meenambakkam Observatory
4. Gemini Studios. 5. Hindu Office.

Full day Excursion: Mahabalipuram.

The Exhibition was open to visitors from 2 p.m. to 4 p.m. on the 26th and from 9 a.m. to 4 p.m. on the 27th and 28th December 1949. Admission to the exhibition was free of charge.

PRAYER

COMPOSED BY
Mahamahopadhyaya Vidyabhushana
PANDIT V. VENKATARAMA SARMA

Sung by Miss S. S. Janaki and Mrs. Parvathi Krishnamurthi

I

दिक्कालादिभिरप्रमेयविभवो यश्चिन्त्यते सर्वतः
दिक्कालोपहितः स्वयं बहुविधोऽप्येको गुणी निर्गुणः ।
नाम्ना चाकृतिभेदतश्च सततं यः कीर्त्यतेऽनेकधा
कूटस्थो निरुपाधिकः स कुरुतात् सर्वात्मको मङ्गलम् ॥

The All-pervading One, whose greatness defies measurement by time and space relationships, yet is always meditated upon by these very notions of dimensional connotation, who, though essentially one, appears as manifold, who, though devoid of attributes, is invested with all of them, who though formless and nameless, is adored in multifarious ways by the imposition of countless names and forms, may such a Spirit beyond the reach of any kind of limitation, the very core of the Universe, bestow auspiciousness on us !

II

यत्सत्तामात्रमग्रे जगदिदमभवद् योऽद्वितीयैकभावः
यो मायी विश्वचक्रं प्रकृतिविलसितं भ्रामयत्यस्तमोहः ।
यः सृष्टौ नाशकार्ये प्रभवति भरणेऽप्यस्य दृश्यस्य देव
तस्मै वेद्याय सच्चिन्मयघनवपुषे सन्तु नः सम्प्रणामाः ॥

May our salutations reach that Universal Spirit, whose very form is Truth, Lustre and Bliss, who in the beginning was the one pure existent from which the whole universe evolved, who with un-erring precision and unlimited energy causes nature to unfold herself, who, equally proficient in creation maintenance and dissolution, is admittedly the paramount controller of the whole of this cosmos.

WELCOME

I

स्वस्ति श्रीः स्वागतं वो गणितपटुधियः ! सङ्गतानां सभेयं
नानादेशेभ्य एवं सकुतुकमिह च श्रेयसे भूयसे स्तात् ।
पाटीशास्त्रप्रमेयोपहितगणितसारोक्तिसद्रत्नमाला-
लङ्कारैर्लासयन्ती लसतु चिरतरं ज्ञानवृद्धान् नितान्तम् ॥

Hail Prosperity!! Oh Scholars, well-versed in Mathematical Science! Welcome to you all. You have assembled here from various places with great joy. May this Conference continue in prosperity and adorn experts in the science of Mathematics with ornaments in the form of the results of the various researches in that science.

II

शास्त्राणां गणिताभिधं सुविदितं शास्त्रं महच्चोत्तमं
यस्मिन् ज्यामिति-गोळ-कालकरणैरुच्चावचे दुर्गमे ।
वैदुष्याहितमेधसां कृतधियामत्रागतानां सतां
प्रोद्यत्प्रश्रयमादरोत्थमधुना चाशास्महे स्वागतम् ॥

It is well known that Mathematics is considered as the most exalted and the greatest of all sciences. This science deals with the various aspects of deep problems of geometry, the planets and astronomy which are very difficult to comprehend. Our most polite and hospitable welcome to the Scholars who are well-versed in the various phases of these problems and whose wisdom has acquired much depth by carrying research work and who have gathered here in the name of Mathematics.

III

अमेयगणितोदधौ करणभेदतोयाकुले
ग्रहादिगणयादसि प्रचुरतत्त्वरत्नोज्ज्वले ।
चरन्त इह धीवराः सुकृतिनो हि पारङ्गता
विभान्तु निजधीप्लवैः सहृदयैश्च सम्भाविताः ॥

May the skilful navigators (धीवर) who are respected by lovers of learning and who, with the aid of the boat of their intellect, have reached the other shore of the ocean of mathematics, which is comprehensive and augmented by innumerable streams (branches of Mathematical Studies) and inhabited by several monsters of the deep (planets, etc.), shine in splendour!

WELCOME ADDRESS

BY

Lt. Col. DIWAN BAHADUR SIR A. LAKSHMANASWAMI MUDALIAR,
B.A., M.D., LL.D., D.Sc., D.C.L., F.R.C.O.G., F.A.C.S., M L.C.,
Chairman, Reception Committee.

LADIES AND GENILEMEN,

On behalf of the Reception Committee and on behalf of the University of Madras, it is my privilege to extend to you a most cordial welcome to this the sixteenth Conference of the Indian Mathematical Society. We are deeply sensible of the honour you have conferred on us by holding this session in Madras. We venture however to express the hope that this honour is due to the part that Madras and the University of Madras have played in the development of your science and in promoting the cause of your Society. The conception of the formation of the Indian Mathematical Society and its establishment in 1907 was due to an alumnus of this University, Mr. V. Ramaswami Iyer who laboured long and untiringly for the success of this Society. The founder and first editor of your Journal was another alumnus of this University, Prof. M. T. Narayana Iyengar who later presided over your Conference in 1930. To another alumnus of this University, my esteemed friend and erstwhile colleague on the Senate, the late Prof. P. V. Seshu Iyer goes the credit of convening the first Conference of the Indian Mathematical Society in December 1916 at Madras. The first President of the Society was the late Mr. Hanumantha Rao, one of the earliest graduates of the University of Madras. Since the inception of the Society, there have so far been fourteen distinguished mathematicians who have been Presidents of your Society and the University of Madras rightly feels gratified that no less than six of these were its alumni, while the seventh Prof. E. W. Middlemast, a Cambridge Wrangler and the third President of the Society, was for long Professor of Mathematics in one of the premier colleges of this University and later its Principal. To crown this record of which any University may well feel gratified, the President-elect of this Conference, my friend Dr. Narasinga Rao is another distinguished alumnus of this University.

But these are not the only claims which South India in general, and the University of Madras in particular lays before you for venturing to extend an invitation to you to hold your session in this city. To mathematicians, not only in India but all over the world, the name of the town of Kumbakonam should be very familiar, not for the cheap jokes indulged in by the vulgar and the ignorant, but as a centre whose chief industry has been the production of mathematicians of repute.

Kumbakonam, however, has even a greater claim to be remembered for ever by mathematicians all over the world, for here it was that the great Ramanujan had his ancestral home and here it was that he had his education,

first in a pial school, as was the custom then for most of us of that generation, and later at the high school, Kumbakonam. It was just 62 years ago, almost to a week that Ramanujan was born and the light that first was lit on that eventful day—the 22nd December—shone forth in later years with a radiating glow, the like of which has not again been revealed in the mathematical horizon during this century. Ramanujan matriculated from this university in 1904 and later appeared for the old F.A. corresponding to the Intermediate Science examination in 1907, but unfortunately failed to secure a pass—an interesting commentary on the efficiency of examinations in general and perhaps some of you may add of the Madras University in particular! I may however state in extenuation and not in justification of the University, that once the genius of Ramanujan was brought to its notice, it tried to make some amends. The discovery of Ramanujan, we owe to members of your Society, to Messrs. Ramaswami Iyer, Seshu Iyer and Ramachandra Rao and to two eminent foreigners, Dr. Walker and Sir Francis Spring. Acting on their advice the University gave a foreign scholarship of £250 per annum to Ramanujan in 1914 which was renewed periodically, the last renewal extending to 1924. Unfortunately his premature demise at the early age of 33 in 1920 deprived the University and the mathematical world of a genius whose potentialities were still but imperfectly realised. To Dr. Hardy, who was his friend, philosopher and guide, we owe a debt of gratitude which can never be repaid fully.

Ramanujan's contributions to mathematics have been but partially made known to the public. The collected papers of Srinivasa Ramanujan edited by Hardy, Seshu Iyer and Wilson form but a part of his monumental works. On his demise, the University has collected all his note books and preserved them in the library. Recently I had occasion to discuss the question of editing and publishing these papers with Dr. S. R. Ranganathan, himself a mathematician and now Professor of Library Science in the Delhi University, and at my request he corresponded with some eminent mathematicians here and elsewhere. Photostat copies of these note books have been taken and it is my hope that ere long, it will be possible for some mathematicians to undertake this task. The University will be glad to do everything possible to have these books edited and published. The photostat copies are shown in the Exhibition upstairs.

Ladies and Gentlemen, if I have ventured to give in some detail the contributions made to your science by scholars from this part of the country, it was not to make any extravagant claim, but merely to show that given the necessary impetus, there is every likelihood of further and more significant contributions being made.

A recent controversy in the political forum seemed to have a very serious and in my opinion a very deleterious effect on the progress of mathematical science in this country—I refer to the controversy over the system of notation and the use of numerals. Apart from the fact that it is generally conceded by most scholars that these were of Indian origin, it seems unfortunate that so

much of passion so much of emotionalism and pseudo-patriotism should have been generated in dealing with a practical issue. Fortunately a wise decision has been arrived at and I trust that at a time when all our energies and all our mental output should be utilised for the extension of the bounds of knowledge in this important science of sciences, such fissiparous tendencies and such bigoted views will not be allowed to invade the realm of science.

Ladies and Gentlemen, we are extremely fortunate in having amidst us one of the greatest physicists to inaugurate this Conference. It is a moot question whether Mathematics has contributed more to development of the physical sciences or whether Physics has helped more the advancement of Mathematics. The truth lies probably between these two extremes and it is a well recognised phenomenon among all scientists that all sciences have their action and reaction on every other science. Sir C. V. Raman has had a long connection with this Society and has sent valuable contributions to the annual reviews and Journal of the society. A dynamic personality, with a flair for calling a spade a spade, and with that genuine love for true knowledge, he has been a great path-finder for all true devotees of science in this country. I have known with what respect and regard he is held in America and Europe and how in many institutes of science, as in the Franklin Institute of Science at Philadelphia, a visitor from India is asked for the latest news and views of Sir C. V. Raman. While I did not hesitate to give the Director the news about Sir C. V. Raman, wisdom no less than prudence made me preserve a discrete silence about his views on any subject, scientific or otherwise. We are extremely thankful to him for having so kindly consented to inaugurate this Conference.

I did not realise that Mathematicians had so much influence that like magnets they can attract stars on the scientific or political firmament of India. That it has been possible for them to get two such eminent Indians as Sir C. V. Raman and Sir C. P. Ramaswami Iyer on the same platform, one to inaugurate the Conference and the other to declare the Mathematical Exhibition open is an achievement of no mean order. But Sir C. P. Ramaswami Iyer's versatility is so all-absorbing and widespread that it did not surprise me to learn that he is to-day the oldest life member of the Mathematical Society. We are glad that he has found time in the midst of a very crowded programme to give us the pleasure and privilege of welcoming him here and requesting him to declare open the Mathematical Exhibition.

Ladies and Gentlemen, we of the Reception Committee are only too keenly aware of our shortcomings and we hope you will forgive us for any inconvenience that may be or may have been caused to you. We take shelter in this fact that this is a Conference of Mathematicians and that the Reception Committee is largely a Committee of Mathematicians. Whether there is any truth in the oft quoted story of Newton and the over boiled egg, it is generally recognised that Mathematicians are so absorbed in the Theory of Numbers or in gazing at the stars that they have little time or inclination to

look to the mundane things of the world. I hope that this fact will serve as an extenuating factor for any lapses on our part and that you will forgive us our defects and accept our assurance that we are delighted to have you with us, on this the second occasion after a lapse of over 32 years when the Conference meets again in Madras.

I have great pleasure in requesting Sir C. V. Raman to inaugurate the Conference.

EXTRACTS FROM SOME OF THE MESSAGES RECEIVED, WISHING SUCCESS TO THE CONFERENCE

[Several messages have been received from distinguished statesmen, public figures, and Scientists in India and abroad wishing the 16th Conference of the Indian Mathematical Society a success. Some of these messages are reproduced below :]

International Congress
of Mathematicians

Harvard University
Cambridge 38, Mass.

It gives me great pleasure to send greetings to the Indian Mathematical Society on the occasion of its Sixteenth Conference at Madras. Indian mathematics is playing a role of steadily increasing importance in world science, and it is a source of stimulus and encouragement to American mathematicians to watch this development.

May I take this opportunity to express the hope that a number of India's leading mathematicians will find it possible to attend the International Mathematical Congress being held at Harvard next year?

GARRETT BIRKHOFF
Chairman, Organizing Committee, International Congress of Mathematicians

♡

Blegdamsvej
15, Copenhagen

Allow me to send my best wishes for the congress and to express my hope or even certainty that the congress with its large and most interesting looking program will be of great value for the development of the Mathematical Science whose roots go back to the oldest history of mankind.

HARALD BOHR

31, Pond Street
Hampstead, N.W. 3.

I wish the Indian Mathematical Society every success in its forthcoming Conference.

JULIAN HUXLEY

♡

John College, Cambridge

I send my greetings and best wishes to the Indian Mathematical Society on the occasion of its Sixteenth Conference.

How fortunate are we Mathematicians to find in the present difficult times a never-ending source of joy and pleasure in the study and advancement of Mathematics! May your Society long and successfully continue to make its contribution to our common welfare.

L. J. MORDELL

♡

Paris

Je salue en la Souiete Mathematique Indienne la vaillante heritiere des traditions qui ont fait de votre pays le berceau de la pensee Mathematique.

Je salue aussi l'union de la Mathematique et de la Physique, symbolisee par le role devolu dans l'inauguration de votre XVI Conference, a l'un des plus grands Maitres de la Physique contemporaine.

HADAMARD

♡

Princeton, N.J.

It is a great pleasure and honour to send fraternal greetings to a gathering of mathematicians in the land of Ramanujan and to wish it success and progress in its lectures and other activities.

Mathematics is the most ancient craft of the mind, always active, and at some times and in some places more so than in others; and it appears that a strong renascence of it is taking place in your country, with the present mathematical generation being as discerning and alert as in any other mathematical community known.

On the face of it there may be some exaggeration in the modern mathematical approach of finding a multitude of definitions, concepts and other structures first, and of examining their intimate relations and the inside contents of their substance only afterwards. But most likely this is only another version of the old Pythagorean dream of encompassing the universe with a structural framework first and then filling it in with geometry, numbers and other like objects afterwards.

So, on to a good meeting,

S. BOCHNER

Oxford

Best wishes to the Indian Mathematical Society for the success of the Sixteenth Conference.

E. C. TITCHMARSH

♡

Edinburgh 8

It is a great pleasure to think of the keen interest in Mathematics which is now so conspicuous in India and the excellent research work that is being published by the Members of the Indian Mathematical Society. May I send my most friendly greetings and good wishes for the success of the Conference.

E. T. WHITTAKER

♡

P. C. Mahalanobis, F.R.S. Statistical Laboratory
Calcutta, 7

I send my greetings to the members of the Conference, and with best wishes for a most successful session.

P. C. MAHALANOBIS

♡

London

I send greetings and good wishes for the success of the Sixteenth Conference of the Indian Mathematical Society.

L. S. BOSANQUET
Hon. Secretary, the London Mathematical Society

♡

Your letter Eleventh December. Unable attend Conference. Please convey my good wishes for its success.

DR. BHABHA

♡

Cordial greetings to the members of the Sixteenth Conference of the Indian Mathematical Society. May the Society continue to render the same high service to mathematics as in the past.

L. E. DICKSON

♡

On the occasion of Sixteenth Indian Mathematical Conference, the Statistical Laboratory, the University of California and Staff wish express admiration for the splendid Research in Mathematics in general and in Mathematical Statistics in particular done in India. Best wishes.

J. NEYMAN

♡

Cambridge, England

Thank you for letting me know of the Indian Mathematical Society's meeting in Madras. It is, of course a pleasure to send you a line with my good wishes.

R. A. FISHER

Ministry of Education, New Delhi

On the occasion of the Sixteenth Indian Mathematical Conference I send you my best wishes for a successful session. I hope the Conference will stimulate study and research in the Mathematical Science in India so that our country may take its rightful place in the rank of countries contributing towards the development of mathematics. India had a glorious past in the field of Mathematics in ancient times and it is my earnest desire that through your efforts it may attain the same reputation again.

Yours sincerely,
TARA CHAND

♡

Council of Scientific and Industrial Research New Delhi

Mathematics is of considerable importance in furtherance of scientific research and India is rightly proud of her mathematicians who have won world wide fame. I hope the deliberations of the Sixteenth Conference of the Indian Mathematical Society will result in greater interest in mathematical studies and research. I wish the Conference every success.

S. S. BHATNAGAR

♡

Apollo Pier Road
Bombay 1

I convey my best wishes for a successful meeting. During six years of war, India was cut off from the great centres of learning in our subject and the development of mathematics in this country could not yet recover from that isolation. Again our research activities are hampered by the economic crisis and by currency restrictions. The access to foreign literature which is a vital necessity for our work and contacts with foreign scholars have become extremely difficult. More importance has to be given to collaboration among the mathematicians of this country. I had the privilege to attend your meetings at Lucknow, Hyderabad, Aligarh, Annamalainagar, Delhi and Waltair and I am sorry not to be able to collaborate with the present Conference. May plenty of good mathematics result from the discussions in Madras.

F. W. LEVI

♡

I need not say how sorry I was not to be able to stay over in Madras for the forthcoming conference of the Indian Mathematical Society. I am sure that the conference will be a success when I see the brilliant band of mathematicians who are organising it. Madras is the province of India where the subject is most earnestly pursued and the present position of Indian Mathematical Society is largely due to Madras enthusiasm.

R. P. PARANJPYE

SECRETARY'S REPORT 1948—'49

On behalf of the Managing Committee it is my pleasant duty to extend a most hearty welcome to all of you who are attending this conference. We thank the authorities of the University of Madras for inviting us to hold our conference here and we are grateful Dr. A. Lakshmanaswamy Mudaliar for his welcome address. We thank Professor C. V. Raman for his opening our conference. Many of us have a vivid recollection of the immense contribution that he made to the success of the last conference at Waltair. We thank Sir C. P. Ramaswamy Iyer for agreeing to open the Mathematics Exhibition.

I regret to report that, since we last met, the Society has lost two members by death. The two we mourn are Mr. Abdulla Butt of the Muslim University, Aligarh, and Rev. C. Pruvot, S.J., till recently, Professor in St. Joseph's College, Tiruchirapalli.

Our first conference was held in Madras in 1916. Since then our Society has grown and gained in strength. At present the activities of our society are along four lines.

- i. Maintenance of a Library and the circulation of periodicals to Members,
- ii. Publication of the *Journal*,
- iii. Publication of the *Mathematics Student*,
- iv. The holding of biennial Conferences.

We are glad to announce that our Society has become a sponsoring Member of the Mathematical reviews. Members of our Society are entitled to obtain Mathematical reviews, at half the usual rates of subscription. We hope that many of our members will make use of this concession to obtain this important publication.

The Library has been till now in the Fergusson College, at Poona. Professor D. D. Kosambi who had been in charge of it for several years past has now resigned his honorary Librarianship. The Society's thanks are due to him for his services as a librarian.

We have endeavoured to maintain regularity of publication of the *Journal* in spite of difficulties of printing and the prevailing high costs. Our thanks are due to all those who have helped the Society by acting as referees of papers submitted for publication.

The Mathematics Student is progressing under the editorship of Prof. A. Narasinga Rao, who has been now elected President of the Society.

We have been holding a conference once every two years. The managing committee will consider at its next meeting the question of annual conferences. I may mention in this connection that what is in effect a Madras

Branch of our Society is functioning in the Madras City under the name of Mathematics Club. The members meet at intervals. I trust that similar regional branches will be formed in some other centres and that it will be possible to publish periodically the proceedings of such gatherings.

During each one of our conferences it is usual to hold a brief and hurried discussion on the syllabuses and teaching of Mathematics. More than once a subcommittee was set up to go into the question of revision of syllabuses but nothing tangible has been done yet. The managing committee will consider this matter and put forward specific proposals about this matter.

We have financial problems to tackle but this is not the time to go into them. We must have our own building for housing the library and we must be able to meet the expenses of the necessary staff (clerical and otherwise) to attend to the work of the Library and to assist those on whom falls some heavy work of the Society.

In conclusion I wish to offer my thanks to the Vice-Chancellor and the authorities of the University for their kind hospitality. I must also thank the Local Secretaries and volunteers for their invaluable assistance.

PRESIDENTIAL ADDRESS

BY

DR. A. NARASINGA RAO

My first duty on this occasion is to offer my thanks to my colleagues in the Indian Mathematical Society for the great honour they have done me by electing me President of this organization. I am afraid that in doing so they have given heed to the promptings of their hearts rather than to their mathematical training, for, any proof of my competence for this high office must rest on many doubtful assumptions and can hardly be considered rigourous. Be that as it may. While well aware of my own limitations both as a man and as a mathematician, I have accepted the offer as giving me an opportunity to serve in yet another capacity an institution dear to my heart and whose growth is indissolubly linked with the progress of our country in matters scientific, technical, and industrial and hence with th well being of the whole community.

We are most grateful to you, Mr. Vice Chancellor and the authorities of the Madras University for your invitation to meet in this great centre of learning and your kind words of welcome. Nearly three decades ago, the Indian Mathematical Society, then barely ten years old, held its first conference in Madras, and now after a long absence it returns like a daughter to her Mother's home with all the joy that such a reunion implies. For, while it is assuredly a

society with an all India membership and organization, it has powerful links with the Madras University and the Madras Presidency. The founder of the Society, the dynamic V. Ramaswami Iyer, whose mathematical enthusiasm remained undamped even by the routine duties of a Deputy Collector, was an alumnus of the Madras University, while another brilliant graduate of the University—Prof. B. Hanumantha Rao was our first President. But the most powerful bond of reverence and affection connecting the Madras University with mathematics—a link which will remain fresh for all time—is that associated with Srinivasa Ramanujan, the greatest mathematician India produced in modern times. It is to the credit of the Madras University that it met an unusual situation with imagination and sympathy and made generous provision for giving his remarkable talents full scope for development and expression. His early death at the age of 32 was the greatest mishap to Indian Mathematics in recent times.

Though Ramanujan is not physically in our midst, one feels that his sprit must be dwelling lovingly in the Research Department created for him, and blessing it; for, it has not done badly, and from its portals have issued forth annually a stream of young men and, I am glad to add, of women also, who have received a training in research methods and are slowly building up by their joint efforts a school of modern mathematics in our country.

The memory of Ramanujan takes us naturally to the Ramanujan Institute which, in a statement made in July 1948, Dr. Rm. Alagappa Chettiar generously undertook to establish and which, I understand, is to start functioning very soon. Indian education owes much already to Dr. Alagappa Chettiar, but in founding an Institute devoted exclusively to Mathematics, he is making history. Any one who runs can see the need for, and the purpose served by, Engineering Colleges and Technological Institutes in the context of to-day, because they lie on the surface. But it requires a rare appreciation of the finer and deeper values of life both intellectual and moral, and of the subtle manner in which the abstract finds its fulfilment in the concrete to realise the importance of mathematics in India reborn. We hope and pray that Karaikudi will develop on generous lines into the Princeton of India, collecting and inspiring talent wherever it may exist, inaugurating a new era of activity at the highest level, and helping to place India on the Mathematical map of the world. I invite you to join with me in rendering our grateful thanks to Dr. Alagappa Chettiar.

I think I have said enough to explain why the Society feels particularly happy in its homecoming to the Madras Presidency and meeting under the auspices of the Madras University. To one like me who believes that nothing is accidental, even the alphabetical kinship between Madras and Mathematics is a matter of deep significance.

This conference has been inaugurated by Prof. Sir C. V. Raman whom we are proud to claim as one of our honorary members, and who is India's finest gift to the world of science. Prof. Raman is not only a brilliant experimental

physicist but is also a competent mathematician whose keen intuitions can penetrate into the reality behind a jungle of mathematical symbols. His presence throughout the Waltair Conference was the major factor which made for its success, as his inauguration to-day is a guarantee of the present session.

Another honorary member of our Society whom we are happy to have in our midst to-day is Prof. Marshall H. Stone of the University of Chicago. The United States of America is to-day the acknowledged leader of the mathematical world and Prof. Stone is no mean person in the mathematical world of America. By his presence and active participation in this Conference, the Indian and American Mathematical Societies have come very close to one another and have brought nearer the day when we shall perhaps have a joint conference of the two societies on the top of some skyscraper in the U.S.A. or perhaps in the beautiful valley of Kashmir. I take this opportunity of requesting Prof. Stone to convey our cordial greetings to the American Mathematical Society and our Admiration for the excellent work it is doing in the cause of mathematics through its periodicals, its fine series of colloquium publications, and through the organization of the first International Congress of Mathematicians after the war to be held in Cambridge (Mass) in 1950.

Another savant from abroad whom we are very happy to welcome to-day is Prof. Herman Wold, the eminent statistician from Sweden who is on a visit to India and has kindly agreed to participate in the activities of our conference. Our thanks are also due to Prof. Tietjens and Prof. Ghatage of Bangalore who are helping the Society to take to wings through their symposium on problems connected with Aeronautics.

I should like also to offer our welcome to Sri C. P. Ramaswamy Iyer the eminent statesman whom we are happy to claim as one of ourselves, and who has kindly agreed to open the exhibition.

You will naturally expect me to say something about the position of Mathematics in India, the needs of the present and plans for its future. There can be no doubt that during the last four decades since the foundation of the Indian Mathematical Society there has been a steady progress from the stage of solving bookish problems by unsuitable methods, to something like a glimpse of what mathematics is and the questions which await study. We are out of the medieval period, but I am not prepared to say that we are in modern times, at least a large part of us. We seem to have got lost somewhere in between. Meanwhile a new situation has arisen, the attainment of independence by India. In exactly a month from to-day, the Indian Republic will be born, and we naturally ask ourselves whether our mathematics is adequate to the situation.

Perhaps some of you wonder what Indian Independence has to do with mathematical standards. Such an attitude betrays an ignorance of what Freedom means. Freedom means, in the first instance, a new poise and outlook marked by dignity and self-respect, and a new orientation of all the values of life, intellectual, aesthetic, moral and material. It means that we refuse to be

judged by any but world standards, and will not be satisfied with a place in the rear. Now mathematics represents, perhaps, the highest efflorescence of the creative human spirit, rivalling the fine arts in its aesthetic appeal, excelling poetry in its condensed richness, and akin to Upanishadic thought in its all embracing abstractness. It is therefore a touchstone of intellectual and moral maturity, and a nation otherwise sound but whose mathematics is not up to the mark is like a good looking and welldressed person but without intelligence or character. I think you will now appreciate why I feel I cannot walk with my head erect in an India which is not mathematically advanced.

The highly abstract character of mathematics might lead one to suppose that, whatever its fascination for a somewhat eccentric class of human beings, it cannot be of much value—at least in its higher flights—in solving concrete problems which face us in modern times. There can be no greater mistake. In fact this highly abstract study appears to have a range of applicability second to none, and the reason for this is very illuminating. The higher and farther away from the *terra firma* of the so-called external world, that Man climbs up the ladder of the Universal, the more extensive is his vision and the more effective his ability to function therein. Let me illustrate by means of two examples from the early history of the human race.

The world of matter impinges on us through a rich diversity of sense impressions, and the aim of both Science and Religion is to discover the fundamental unity which underlies this diversity of appearance. We soon perceive that it is convenient to classify the world into objects, and these again into classes each having some common characteristics while differing widely in detail. Thus are born universals like "man", "tree" etc., and at higher and still higher levels ideas like "redness", "beauty", "truth" and so on. Each new concept thus coined is a victory of mind over matter and is celebrated by a Namakaranam—the imprisonment of the concept inside a name.

We are witnessing the stage of human evolution which made possible "speech"—one of the major achievements of the human race. Without the power of abstract thought necessary for concept-building, this would have been impossible. Another capacity which man developed simultaneously, or even earlier, was his ability to use symbols, i.e. to represent a thing by something else to take its place for certain purposes. Early man had artistic talents, and the earliest symbols were either pictures or models or something intimately connected with the object represented. The symbol was more easy to handle, and for some purposes better than the object. For example, one could safely vent one's anger on the symbol while being careful not to approach within a mile of the original. This symbolic stage of the evolution of the human mind has given us "writing" where conventional symbols are used to represent speech sounds, and the converse operation of "reading." Just imagine, if you can, what mankind would have been without speech and writing, and you can realize how much we owe to the power of abstraction implied in concept-building and symbolism.

In the next level of evolution of the human mind, which I shall call the mathematical level, both concept-building and symbolism are carried to unheard of heights of achievement, but in addition there is the emergence of a new factor of the utmost importance. Till now it has been a study of quality. This new factor is "quantity". A comparison of two individual material objects gives rise to the relations of bigger and smaller and ultimately to the measurement of continuous magnitudes, while a comparison of two groups of objects leads to the concepts of counting and the number system. These two aspects of quantity—the discrete and the continuous—leading respectively one to Arithmetic and the other to Algebra and Geometry, must have developed simultaneously, and independently. It was given to the mathematicians of the nineteenth century, Cantor and Dedekind to weave these into a single beautiful discipline using rational numbers as a bridge. The introduction of quantity (or measurement) leads naturally to the study of the functional relationship between changing quantities—the search for the permanent in a world of change-a quest which reminds us of the goal of Religion. Like the evolving human mind, all observational sciences recapitulate the concept building and symbolic stages, but they will be considered to have attained maturity only when they have passed over to one where measurements are made and relations between measured quantities discovered. Psychology and the Social sciences are in the beginnings of this stage of development. By providing a calculus and a technique for dealing with measured values and a means of discovering relationships, Mathematics acts as the foster mother of young branches of knowledge.

Besides Arithmetic and Algebra which deal with the discrete and the continuous variable, Mathematicians in various countries like Babylon, Egypt, India had studied the measure relations of various geometrical patterns and obtained a number of interesting empirical results. It is to the everlasting credit of the Greeks headed probably by Euclid that they were the first to study the dependence of the results on one another, and to cast Geometry into a logical mould where a majestic structure arose gradually from a few apparently innocent, and simple set of postulates. It was almost like getting a beautiful building, for not even a song, but the utterance of a few platitudes! We owe to Euclid two priceless gifts to mathematics—one a working model of the structure of Physical space which has stood all the demands made on it by the Engineers, Physicists and Astronomers except in the region of the infinitesimally small and the infinitely large, the atomic and the cosmic scales. The second gift of Euclid is a pattern for a mathematical structure, erected from expressly stated independent postulates. Though Euclid's structure has had to be strengthened by Hilbert and others, the ideal which Euclid had in view has not been essentially modified. On the other hand this ideal of Mathematics as a self-consistent logical thought-structure has permeated every branch of mathematics, and to-day the mathematical shop window is full of strange models of entrancing beauty and tenuity, while others are being manufactured, either in order to meet particular requirements, or for their own intrinsic

elegance. You can have an algebra where numbers do not come in at all and $A+A=A$, another in which $AB \neq BA$, while if you want a geometry with a finite set of points and lines having the characteristic properties of ordinary geometry, you can certainly have one based on a Galois Field. The order and continuity properties of the linear continuum have been analysed and dissected, and each component generalized and developed by itself or in combination so that, the continuum has a number of topological grandchildren which it can hardly recognize. You can have geometries where lengths behave like angles, and angles like lengths; and others where parallel lines do not exist at all, or there are an infinity of them through a given point; still other spaces in which you can find your way about but cannot say how far you have travelled,—the maddest collection of self-consistent structures that one could dream of. And they are all necessary because all sorts of people walk in and make strange demands—the theoretical physicist wants a model which can be interpreted sometimes as a particle and sometimes as a wave, the relativist a suitable symbolism for invariance in form under arbitrary transformations, the psychologist a means of analysing the separate effects of several factors whose combined effect is known, and the stastistician a means of eliminating chance variations from his conclusions based on particular samples, while the Vedantin wants an elucidation of some obscure upanishadic passages. It is because mathematics has freéd itself from the study of any particular category of sense experiences, in sharp contrast to the other sciences, and functions at a higher level of abstraction that it has been able to meet these varied demands, and is able to provide an armoury of weapons suitable for the various sciences.

In the stress of modern life and an age of keen competition for quick results, there is always the danger that fundamental studies may be neglected in favour of applied studies like engineering and technology which are directly connected with economic well-being. The temptation will be all the greater in a country like ours which is trying to compress half a century of progress into a decade, but it must be firmly resisted. The applied sciences are like the colourful flower and the luscious fruit, but they must be supported by the stem of pure science through which flows the sap of research, and this again will wither away unless the root of mathematics which supports the whole plant is well watered and kept in a healthy condition. Old Aesop when he wrote about the danger to the goose that laid the Golden egg was making a statement true for all time. Mathematical studies have always reached a high level of development in those countries where intellectual and industrial progress has been very marked, for example, in Greece in the heyday of its glory, in Germany before Hitler and in the United States of America at the present day. It used to be said in the old days that the industrial progress of a country could be judged by the amount of sulphuric acid it consumed. I do not know what the corresponding raw material is to-day—probably electricity. In the same way, the scientific progress of a country may be judged by the amount of mathematics it consumes. The surest way to commit national suicide is to neglect mathematics.

Even the U.S.A., advanced as it is in so many respects, found under the crisis of war, that a bottleneck in the war-effort arose not in regard to material or equipment, but in the mathematical training of its youth, and feverishly set itself to remedy the defect. We do not want war, nor need we wait for a crisis. But we have to recognize that Mathematics is needed both in peace and in war, and that an inadequate standard in mathematics means pernicious anaemia in all the sciences both pure and applied. Again we anxiously ask ourselves the question : Is our Mathematics of the proper standard ?

I am not prepared to answer in the affirmative. Our teaching is still formal and lifeless, our standards poor, and the number of our advanced workers ridiculously small for a country of our size and with our past history. Our syllabuses are antiquated and a student can take his highest University degree in Mathematics without knowing anything of modern topics. Among the teachers even in the bigger colleges and University centres, scholarship is rare and research if there is any at all, confined to a few. This sorry state of affairs has got to be changed, and the drive for this must come primarily from the teachers themselves, an organization like the Indian Mathematical Society acting as the coordinating agency to canalize these activities. To bring about this much-needed and urgent reform, we have to begin right at the first rung of the educational ladder. Taking the primary stage, one has to see whether by the use of play methods and the elimination of unnecessary drill something more could be achieved in this period than at present. At the secondary stage we have to organize mathematics as a laboratory study on lines similar to the physics Laboratory, where pupils experiment with numbers and figures and finally strengthen their convictions by a formal proof. An attempt must be made to teach the elements of trigonometry, analytical geometry and calculus at this stage as is done in other countries. Coming to collegiate mathematics the same attempt at including more and more so as to bring the highest university examinations close to the research front will be necessary. There is need for periodical conferences on the teaching of mathematics to discuss the efficiency of different modes of presentation and the results of experiments in pedagogy. These gatherings will also serve to place before teachers ideals of scholarship and efficiency. No scheme like the above is likely to succeed unless we educate the teacher also at the same time by means of refresher courses, travel facilities to visit different parts of India to benefit by watching the teaching of experienced teachers, and similar methods.

I have remarked that the Pure Mathematician is a Brahma creating thought forms of entrancing beauty at a level higher than that of sense experience and using a language which evolved at a later stage of evolution than ordinary speech ; that his shop window contains many curious models and that all kinds of persons approach him and make strange requests which he tries to meet. Let us now watch the Vedantin walk in and ask if some obscure passages in the Upanishads which

transcend ordinary logic, can receive clarification at his hands. Take for example the description *Anoraniyan Mahatomahiyan*. How can the same entity be both smaller than the smallest and bigger than the biggest? Now it happens that Mathematics has independently arrived at precisely an identical situation, for while a line of infinitesimal length, say a thousandth of a millimeter, is microscopically small compared with say the distance to the Andromeda Nebula, Cantor has shown that the number of points on the one must be considered equal to those in the other, in the sense that a one to one correspondence may be established between their points. Again in Transfinite Arithmetic we meet with a situation which is appropriately described by the sloka "Purna madah purnamidam purnath purnamudachyate, purnasya purnamadaya purnameva avasishyathe" "that one is whole, this too is whole, and from the whole is born. Take away whole from whole, what remains is whole" It is said that creation is "ichamatram", it is merely a matter of wishing and the One separates itself into a myriad of different entities, and that then begins a long and slow process of integration by which the basic unity which underlies this apparent multiplicity asserts itself and ultimately realises its oneness. This is Iswara's Leela, of the evolution of the Universe. The mathematician plays a similar leela in Analytical Geometry. Take the plane. It is complete and without recognisable parts and no point can be distinguished from another. Now introduce the x-and y-axes, and the plane now gets broken into a myriad of distinct points any of which can be specified by its coordinates different for different points. Then begins a long process of integration, the points being first grouped into curves like straight lines, circles etc.; these again are grouped into classes like circles of equal size, ellipses with the same principal axes etc., which are equivalent under the Euclidean group of congruent transformations. Relatively to this order of ideas, two circles with different radii are different, but if they have the same radius they are identical. A higher group, like the group of projective transformations obliterates the distinction between the circle, ellipse, parabola but curves of different degrees are to be considered different from one another. If we pass on to continuous transformations the topological group—all distinctions between simple curves vanish, but closure is an invariant property and we recognize only two categories namely closed and open curves; while when discontinuous transformations are also admitted, all categories merge into one, and the only property which survives is that of existence or non-existence. Is not this situation similar to what the Vedanta teaches, that identity or difference depends on the level from which we view it, that distinctions which are relevant at the Vyavaharika level lose their significance at the level of Realisation. For the cricketer the ball is a hard spherical object which offers resistance to a bat, while for the physicist the ball is mostly empty space traversed by a swarm of molecules in rapid motion. Which of these is the correct picture? Each is correct at its own level, and the wise man is he who can recognize when the different orders of ideas are valid

The equality of all men under democracy is not inconsistent with the choice of a particular person for a post, or for marriage with one's daughter, rejecting all the others. The Jivanmuktha is he who can function at ease at all levels - who can enjoy a hearty meal and joke and merrymake with others, and can with equal ease lose himself in the vastness of the Divine Love and Grace.

I have tried in this address to give an idea of the place of mathematics in the evolution of the human mind and of science, its importance in the make up of present day civilization and the wide diversity of situations it is competent to meet. India has been the Land where Mathematics has always been honoured, for has it not been said:

'Yatha Sikha mayuranam Naganam phanayo yatha
Thadwat vedanga sastranam ganitham murdhani sthitham"
May it ever continue to flourish in this land.

ABSTRACTS OF PAPERS

SECTION A

Modern Algebra and Abstract spaces

K. VENKATACHALIENGAR, Mysore

An investigation of the Lie and Jordan algebras connected with the orthogonal group in three and four dimensions

In this paper we consider the Lie-Algebra generated by three symbols a, b, c, satisfying relations $bc - cb = a$, etc. as well as the corresponding Jordan-Algebra with the relations $bc + cb = a$, etc. Recently Jacobson (*Amer. Jour. Math.* Jan. 1949) has proved that if any one of the three symbols of the Lie algebra satisfies a polynomial relation the corresponding algebra is finite dimensional. In this paper it is proved that the result is true in the case of the Jordan algebra also, and the exact polynomial relations that can occur are also obtained. After the choice of a suitable basis the resolution of these algebras into direct sums of minimal ideals has also been accomplished.

Further it is proved directly that every finite dimensional representation is completely reducible. The structure of a representation of the Jordan algebra has also been obtained.

Since the Lie-Algebra of the orthogonal group in four dimensions (i e. with six symbols) is the direct product of two such algebras with three symbols, these results have also been extended to four dimensions. Further the corresponding Jordan-algebra with six symbols has also been resolved into direct sums of minimal ideals.

K. PADMAVALLY, Madras

Weakening the topology of a connected Chain

Let C be a chain connected under its intrinsic topology ϕ; and ϕ_1 a topology of C weaker than ϕ. Let P be the set of points at which ϕ is equivalent to ϕ_1, Q the set at which ϕ_1 is weaker than ϕ, T_1 the set of points which determine the section classes into which C is disconnected under ϕ_1, and T_2 the set of points

at which there is local disconnection under ϕ_1. Then the following results are proved.

$$Q \subset T_1 \cup T_2 \subset Q$$
$$T_1 \subset Q$$
$$D(T_1) \subset T_1 \cup T_2.$$

V. K. BALACHANDRAN, Madras

On a Problem of Birkhoff

In his recent paper "Representation of lattices by sets" (*Trans. Amer. Math. Soc.* Vol. 64. (1948) pp. 299-316) Birkhoff has stated 'The question remains open whether, in a complete lattice satisfying some infinite distributive law, completely meet-irreducible dual ideals are completely prime.'

Our object in this note is to show that the answer to this problem of Birkhoff is in the negative. More precisely, it is shown that in any complete infinite Boolean algebra there exists at least one completely meet-irreducible ideal (of either species) which is not completely prime.

M. V. SUBBA RAO, Madras

On a eigen value property of linear operators in a Banach Space

An eigen λ is said to be of order N of a linear operator T if $(T-\lambda)x=0$ for $x \neq 0$, and $(T-\lambda)^N x=0$ has more solutions than $(T-\lambda)^n x=0$ for $n \leqslant N$; but has the same number of solutions as $(T-\lambda)^n x=0$ for $n \geqslant N$. If D_n denotes the closed set of all elements x for which $(T-\lambda)^n x=0$, then the following results are proved:

(1) if T maps a Banach Space E on to E, then either λ is not an eigen value of T, or it is an eigen value of order infinity, and $D_1 \subset D_2 \subset D_3 \subset \cdots$

(2) if T maps onto $E_1 \subset E$, and if $(1-T)$ is totally continuous, then either λ is not an eigen value of T, or it is an eigen value of finite order, say N: and if $T^n(E)=E_n$, then $E_1 \supset E_2 \supset E_3 \supset \cdots \supset E_N = E_{N+1} = \cdots$

V. RAMASWAMI, Waltair

On a theorem of Gelfand and Hille

This paper deals with an interesting aspect of the well known theorem of Gelfand and Hille on necessary and sufficient conditions for a generalized nil-potent of a Banach space to be a nil-potent of specified order.

K. G. RAMANATHAN, Princeton N. J.

Discrete groups on Symmetric spaces

Let Z be a complex matrix of m rows and n columns and let h be the hermitian matrix $h = E - Z'\bar{Z}$ (Z' is transpose of Z and $\bar{Z}$ is conjugate of Z). Consider the bounded domain P

$$P : \{Z \mid h > 0\}$$

in the space of m, n complex variables Z_{kl} ($k = 1, \ldots m$; $l = 1, \ldots n$). It is known from a theorem of E. Cartan that the most general analytic homeomorphism of P on to itself is a connected Lie group G defined by the transformations

$$Z \longrightarrow (AZ + B)\ (CZ + D)^{-1}$$

where A, B, C, D are properly chosen complex matrices. We show that G has discrete subgroups which have a discontinuous representation in the sense of Poincaré in P. One such subgroup is obtained by restricting the elements of A, B, C, D to integers of an imaginary quadratic field. Another example can be furnished from the theory of units of hermitian forms by a method similar to Siegel's. If we construct fundamental domains F for these subgroups we find that $\bar{F}$ is compact in the second case.

That the space P is a generalisation of the unit circle is obvious by putting $m = n = 1$. In the case $n = 1$ the space P is the interior of the complex sphere C

$$C : 1 - (Z_1\bar{Z}_1 + \cdots + Z_m\bar{Z}_m) > 3$$

Discrete groups (actually, only one example) were constructed on C by Picard ($m = 2$) and for any m by Giraud and Fubini. The interesting fact about the space P is that for $n = m$ the transformation

$$Z = i\ (E + Z_0)\ (E - Z_0)^{-1}$$

where $P : E - Z_0'\bar{Z}_0 > 0$ carries P into the space $i^{-1}\ (Z' - \bar{Z}) > 0$ in the space of n^2 complex variables. This new space is a generalisation of the notion of upper half plane. For discrete groups on this space we construct fundamental domains and show that their volumes are finite.

SECTION B

Arithmetic and Theory of Numbers

D. R. KAPREKAR, Devlali

Studies in Demlo Number theory

In three papers the author obtains various results connected with his Demlo Numbers. In paper (i) are considered sets of numbers as in Tarry's problem with the further property that the digits of the numbers on each side written in succession are reverses of each other. Thus

$$4^n + 20^n + 42^n + 3^n + 41^n + 25^n = 5^n + 21^n + 43^n + 2^n + 40^n + 24^n \quad (n = 1, 2, 3)$$

Rules are given for constructing such "reversed power series" for any value of n.

Paper (ii): Take a number of N digits with the digits in increasing order of magnitude, reverse the digits and substract the former from the latter, thus getting the number N_1. If this process be iterated, it is shown that certain successions of digits are repeated. Such "rotative series" of digits are obtained for all values of N from 1 to 12. In paper (iii) are considered the trisectible structure of the products of Wonderful Demlo Numbers.

M. S. SRINIVASAN, Tanjore

Ratios of power period-numbers

If $n < d$, and d is prime to 10, then the recurring part of the decimal representation of n/d, is called the period number of n/d. Thus the period number of 12/91 is 131868. In this paper are discussed the ratios of the period numbers of $1/d^r$ and $1/d^s$, the ratios of the period numbers of $1/dd'$ and of $1/d$; divisions and multiplications involving period numbers, and the evaluation of products like $(3)_m(7)_n(1)_p \times (7)_m(1)_n(3)_p \times (1)_m(3)_n(7)_p$ where $(7)_n$ means the period number of 7/9 repeated m times.

A. A. KRISHNASWAMY AYYANGAR (Waltair) and
A. K. SRINIVASAN, (Mysore)

Some Properties of a Class of Integers

In June 1948 (*Current Science* 17, 179–180) one of the present writers announced a class of integers, named 'Practical Numbers'

which we now call more briefly P-numbers. They possess the property that every number less than a P-number is either itself a divisor or the sum of distinct divisors of the P-number.

The following additional properties deserve notice :

(1) Every P-number must be of the form $12m$, $12m+4$, $12m+6$, or $12m+8$. Every number of the form $2^{\alpha} 3^{\beta}$ ($\alpha>0$, $\beta\geqslant 0$) is a P-number.

(2) If n is a P-number, nn' is also a P-number if $1\leqslant n'\leqslant 2n$. Dr. T. Vijayaraghavan with whom one of us discussed the paper gave the best inequality in the form $1\leqslant n'\leqslant f(n)$ where $f(n)$ is the least positive integer which is neither a divisor of n, nor the sum of distinct divisors of n. We are also indebted to him for the basic theorem.

(3) If n be a P-number with p as its highest prime divisor, then n/p is also a P-number.

(4) If n is a P-number, then the necessary and sufficient condition that np^{α} (when p is any prime) is also a P-number is that np is a P-number.

(5) If $2^{\alpha}p$ is a P-number (p being an odd prime), then $2^{\alpha}p_i^{\alpha_i}p_j^{\alpha_j}\cdots$ is also a P-number, if p_i, p_j, $\cdots$ are all odd primes not exceeding p.

It is conjectured that the number of P-numbers not exceeding $x(>2)$ is always greater than the number of primes not exceeding x and that the n-th odd prime is also always greater than the n-th P-number.

A Table of P-numbers $\leqslant 1000$ is appended to the paper.

HANSRAJ GUPTA, Hoshiarpur

On a Problem of Erdos

In this paper, Lambert's solution of the equation

$$\phi(x)=k! \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

is obtained by an independent method. The method is then used to prove a conjecture of Klee. If $S_i(k)$ denotes the number of solutions of (1) in M_i, where M_i denotes the set of all integers m_i in the canonical factorization of which exactly i distinct primes occur to the first power, then

$$S_1(k)\geqslant 1, \text{ and } S_1(k)\to\infty \text{ as } k\to\infty.$$

M. VENKATARAMA IYER and M. BHIMASENA RAO, Bangalore

Types of solutions of $x^3+y^3+z^3=1$ in integers—Part 4.—

[The first two parts have been published in the J. I. M. S. and the third part was read at the Annamalainagar conference.]

Taking as usual, $\dfrac{1-z}{x+y}=t$, the type in this part, a general solution of problems of type 19 has been given. Choosing a and b so as to make $w=-24a^2+178ab+51b^2=1$, the corresponding values

$$x=480a^2+250ab+123b^2,$$

$$y=-472a^2-230ab-120b^2,$$

$$\text{and}\quad z=176a^2+202ab+6b^2$$

will give the solution. With the same set of values for w and x, a second set of values for y and z are:

$$y=-492\ a^2-242\ ab-118\ b^2$$

$$z=-204\ a^2-26\ ab+44\ b^2$$

Sets of numerical values are also furnished taking $a=27179$ and $b=3533$.

A. V. PRASAD, Patna

On a result of Prof. H. Davenport

Prof. H. Davenport recently proved the following theorem. Let $f(x, y)=ax^2+bxy+cy^2=a(x+\theta y)(x+\theta' y)$ be an indefinite quadratic form with real coefficients. Suppose that $a\neq 0$ and that θ and θ' are different irrational numbers. Then there exist two real numbers p and q, such that

$f(x+p,\ y+q) > \dfrac{1}{128}\ \sqrt{d}$ for all integral x, y where $d=b^2-4ac$. He further applies this to prove that Euclid's alogorithm is not valid in K $(\sqrt{m})$ if

$$m > 64^2,\quad m\equiv 2 \text{ or } 3 \pmod 4$$

$$\text{or if } m > 128^2,\ m\equiv 1 \pmod 4$$

The object of the present paper is to replace $\dfrac{1}{128}$ by $\dfrac{1}{36}$ by certain improved methods. The result is being improved.

F. C. AULUCK, Delhi

Certain new types of partitions

We consider in this paper partitions of integers into summands when each summand s is taken to be s-fold: e.g. 3 can occur in three ways as 3_1. 3_2, 3_3; 5 can occur in five ways as 5_1, 5_2, 5_3, 5_4, 5_5. Such partitions are of interest in Statistical Thermodynamics. If $P(n)$ denotes the number of such partitions of n, then it is easily seen that $P(3)=6$, $P(4)=13$. It is shown in this paper that

$$P(n) \sim \frac{C}{n^{25/36}} \exp. \left[\frac{3\alpha^{1/3}}{2^{2/3}} n^{2/3} \right]$$

$$\text{where } C = \frac{e^{\beta} \alpha^{7/36}}{2^{11/36} (3\pi)^{1/2}};$$

$$\alpha = \zeta(3), \quad \beta = \zeta'(-1).$$

C. S. VENKATARAMAN, Trichur

On the indices of Ordinal Functions

It is shown by the author [*Vide J. I. M. S.* **10**, 1–12, (1946)] that the result of convolving an Ordinal function

$$\psi(M_1, M_2, \cdots, M_r, N_1, N_2, \cdots, N_s)$$

which vanishes whenever any $N_i >$ any M_j, ($i=1, 2, \cdots, s$ and $j=1, 2, \cdots, r$) is a multiplicative function $f(M, N)$ which *vanishes whenever* $N^r > M^s$, (i.e.) whenever $N^{r/s} > M$. This enables us to associate with an Ordinal function $f(M, N)$, *an index* which is a *positive rational number* and thus suggests a further extension of Ordinal functions considered in my paper '*The Ordinal Correspondence etc.*' [*J. I. M. S.*. **10**, 81–101, (1946)] where the indices are *only* positive *integers*. In the present paper, is developed a theory concerning multiplicative functions $f(M, N)$ which vanish whenever $N^k > M$, where k is any *real* number, thereby giving a complete generalisation of Ordinal functions by means of an index.

M. V. SUBBA RAO, Madras

On the representation of a number as a sum of two squares.

The following results are obtained by equating coefficients in certain theta function expansions:

1. The number of ways of representing a number n as a sum of two triangular numbers (*i.e.*) numbers of the form $n(n+1)/2$ is

$$\sum (-1)^{(d+d'+2)/4},$$

summed over numbers d such that

$$dd' = 32n+8, \; d+d' = 2 \bmod 4.$$

If n is a square, subtract unity from the above expression.

2. The number of ways of representing n as a sum of two (non-zero) square numbers is

$$\sum (-1)^{(d+d'-2)/4}$$

summed over d such that $dd' = 16n$, $d + d' = 2 \pmod 4$.

3. Let $R_1(n)$, $R_2(n)$, $R_3(n)$, $R_4(n)$ denote the number of representations of n as respectively $(4a)^2 + b^2$; $(4a+2)^2 + b^2$; $(4a+1)^2 - b^2$, $(4a+3)^2 - b^2$, where in the first two $a > 0$, $b > 0$; in the third $a > 0$, $b \geqslant 0$; and in the fourth $a \geqslant 0$, $b \geqslant 0$, then

$$R_1(n) - R_2(n) = R_3(n) - R_4(n).$$

If n is an odd square, add to the right side $\frac{1}{2}[1 + (-1)^{(n+1)/2}]$, and if n is an even square add $\frac{1}{2}[1 + (-1)^{n/2}]$.

SECTION C

Analysis

D. K. SEN, Kolhapur

Application of Newton's method of approximation in the case of nearly equal roots

B. N. PRASAD and J. A. SIDDIQI

On the Nörlund Summability of the derived Fourier series

1. Let $f(x)$ be a function of the class $L(0, 2\pi)$ and periodic with period 2π. Let the Fourier series associated with $f(x)$ be

$$\tfrac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

The r-th derived series would be

$$\sum_{n=1}^{\infty} (-1)^{r/2} \, n^r \, (a_n \cos nx + b_n \sin nx) \qquad (r \text{ even})$$

$$\sum_{n=1}^{\infty} (-1)^{(r-1)/2} \, n^r \, (b_n \cos nx - a_n \sin nx) \qquad (r \text{ odd}).$$

If at a point x the function $f(x)$ satisfies the relation

$$\tfrac{1}{2}\{ f(x+t) + (-1)^r f(x-t)\} =$$

$$\sum_{i=0}^{[r/2]} f^{(r-2i)}(x) \frac{t^{r-2i}}{(r-2i)!} + w(x, t) \frac{t^r}{r!}$$

where $w(x, t) \longrightarrow 0$ as $t \longrightarrow 0$, then $f^{(r)}(x)$ is called the r-th generalized derivative of $f(x)$ at the point x.

The (C, k) summability of the r-th derived series has been considered by Zygmund whose theorem was subsequently generalized by Wang.

In this paper we give an analogous theorem for the Nörlund summability which, in view of the fact that the Cesàro summability is a particular case of Nörlund summability, will include the theorems of Zygmund and Wang as special cases. The theorem we prove is as follows :—

THEOREM. *The r-th derived series of the Fourier series is summable (N, p) to $f^{(r)}(x)$ at every point x at which*

$$\int_0^t |w(x, u)| du = o(t),$$

provided (i) $n^j |\Delta^{j-1} p_n| = O(P_n) \qquad (j=1, 2, \ldots, r+1)$

(ii) $\sum_{k=1}^{n} k^{j-m-1}(n-k)^m |\Delta^{j-1} p_k| = O(P_n)$

$(j=1, 2, \ldots, r+1) \qquad (m=0, 1, \ldots, r)$

(iii) $\sum_{k=1}^{n} k^{-r-1} | P_k | = O(P_n/n^r).$

S. M. SHAH, Aligarh

The Maximum Term of an Entire Series (VI)

This paper is in continuation of my previous note (*Math. Student* 10 (1942) pp. 80–82; *Quarterly Jour. of Maths.* (Oxford) 19 (1943) pp. 220–223). I prove here that

Given ρ $(0 \leqslant \rho \leqslant \infty)$ and any increasing function $\psi(x)$ such that

$$\limsup_{x \to \infty} \log \psi(x)/\log x \leqslant \rho$$

there exists an entire function $f(z)$ of order ρ such that

$$\liminf_{r \to \infty} \psi(r) \log M(r)/\nu(r) = 0$$

$$\limsup_{r \to \infty} \log M(r)/\nu(r) \log r = 1.$$

R. S. VARMA, New Delhi

On a Generalisation of Laplace's Integral

At the last Science Congress held at Allahabad, I gave a generalisation of Laplace's Integral

$$f(x) = \int_0^\infty e^{-xt}\,\phi(t)\,dt.$$

by means of Whittaker's confluent hypergeometric function. Since then, two inversion formulae have been given by Mr. K. M. Saxena and three by myself. Recently Mr. K. P. Bhatnagar has deduced certain interesting properties. In the present paper I have investigated some results which extend further the theory of the generalised Laplace's Integral.

SECTION D

Geometry

C. N. SRINIVASIENGAR, Bangalore

On the Serret-Frenet formulae in Riemannian Geometry

Serret-Frenet formulæ in Riemannian Geometry were first introduced by Blaschke: *Math. Zeit.* Vol. 6 pp. 94–99, and are explained in text books, Vide: Eisenhart *Riemannian Geometry.* The treatment however appears very artificial, unlike the elegant and natural treatment that one is familiar with in ordinary three-dimensional geometry. The present paper is an attempt to give a more natural proof, by introducing a set of osculating Euclidean spaces at any point of a curve in general Riemannian Space.

Miss MANIKARNIKAMMA, Bangalore

The Spherical Bertrand curve

A Bertrand curve in ordinary 3-space which lies on a sphere is found to satisfy the equation $\rho = a \sin \frac{S}{k}$ where S is the arc of the conjugate Bertrand curve. In particular, this leads to the result that the spherical curve of constant torsion is given by $\rho = a \sin \frac{s}{k}$.

P. B. BHATTACHARYYA and RAM BEHARI, Delhi

Some Properties of the Skewness of distribution of the Generators of a Ruled Surface

In a paper published in the *Bull. Cal. Math. Soc.* 37 (1945) p. 133 V. R. Chariar has defined the skewness of distribution of the generators of a ruled surface and has obtained some properties of it. In a subsequent paper published in *Proc. Ind. Acad. Sci.* 30 (1949) p. 49, he has obtained more properties of it. The object of this paper is to obtain some further properties of the skewness of distribution and to extend the application of the concept to the ruled surfaces through a line of a rectilinear congruence.

N. PADMA, Madras

Extension of Dupin indicatrix to affine space, A_n

The Dupin Indicatrix at an ordinary point O to a surface S_2 in a Euclidean space explains the existence of "conjugate directions" at O.

In this paper it is shown that in an affine space A_n, with the symmetric "affine connection" Γ^i_{jk}, the indicatrix at an ordinary point O to a surface S_{n-1}, should be considered as the relationship between pairs of directions OP and OP′ on the surface, such that the result of infinitesimal parallel displacement of OP along OP′ will still give a tangent line at P′. The form of the theory is preserved in A_n, as it is shown that this relation between OP and OP′ is symmetric.

We can derive from this theory the condition for a covariant vector field in A_n, to build up a one-parameter family of $(n-1)$-dimensional surfaces or to have the "constitutive dimension $(n-1)$", or to be "X_{n-1} - bildend".

K. RANGASWAMI AIYER, Annamalainagar

On the structure of Joachimsthal's circle system

It is shown in this paper that the totality of Joachimsthal's circles of a conic, constitute in circle space an elliptic ruled quintic surface whose double curve consists of two generators.

M. VENKATARAMAN, Annamalainagar

Lattice structure of projective planes

The sub-planes of a projective plane form a lattice with respect to set inclusion. This lattice is a matroid lattice if the projective plane is a Desarguesian plane.

θ - logique triangles

If the θ - perpendiculars from the vertices of a triangle Δ_1 to the corresponding sides of a triangle Δ_2 are concurrent, Δ_1 is said to be θ - logique to Δ_2. Some properties of triangle pairs having this relation are studied in this paper.

SECTION E

Statistics, Applied Mathematics, History of Mathematics

M. LAKSHMANAMURTI, Waltair

Generalization of a problem occurring in skewness and kurtosis

The paper deals with the generalization of inequalities and extrema connected with α_m for general values of m. The cases for $m=3$ and $m=4$ are already known.

A. A. KRISHNASWAMI AYYANGAR, Waltair

The frequency distribution of Indian Mathematical productivity as exhibited in the Journal of the Indian Mathematical Society and the Mathematics Student

In a stable economy i.e. one not associated with revolution or war, it has been stated by Pareto (1848–1923) that the distribution of income in all places and at all times will be given approximately by the empirical formula

$$y = ax^{-v}$$

where y is the number of people having the income $\geqslant x$ and v is approximately 1·5. If appears that any measurable special ability, like playing billiards, also comes under this law. Indeed, other abilities which are rare also come under this law as showr by

A. J. Lotka who spoke of the inverse square-law of scientific production. Recently Arnold Dresden, in his report of the Scientific work of the Chicago Section of the American Mathematical Society 1897–1922 studied the frequency distribution of the papers presented at this Section during a quarter of a century and H. T. Davis has obtained a parabolic curve to fit this distribution in the form

$$\log y = 3{\cdot}74877 - 2{\cdot}11012 \log x,$$

where y is the number of persons contributing more than x papers $(x > 3)$

In this paper, a study is made of the number of papers actually published in our journals during the period of 40 years running from 1908 to 1947 and the curve fitted by me to this distribution is given by

$$\log y = 3{\cdot}5163 - 1{\cdot}7576 \log x$$

It is interesting to remark that our distribution bears a very close analogy to the American distribution and the unfortunate corollary is perhaps that we are a quarter of century behind in the tempo of mathematical productivity.

It seems possible to grade mathematical ability into three classes A, B, C whose frequencies are in the ratio of 1 : 3 : 9 by either of the two following criteria :

(*a*) pages contributed: A (above 80), B (21 – 80), C (less than 21)

(*b*) Number of contributions: A (above 12), B (4 – 12) C (less than 4)

The Coefficient of mean square contingency between the two gradings is as high as ·8 and χ^2 – test reveals that they are not significantly different.

A. S. KRISHNAMOORTHY, Madras

A Multivariate Gamma Type Distribution

The variance in a sample of given size n from a normally distributed population has a distribution of Type III; we investigate the distribution of the variances of n variates in samples from n-variate distribution.

Starting with the n variate normal correlated distribution in $x_1, x_2, \cdots, x_n$, it is shown that the substitutions $x_i^2 = 2\xi_i \ (i = 1, 2 \cdots, n)$

lead to the distribution function whose moment—generating function $G(\alpha_1, \alpha_2 \cdots, \alpha_n)$ is

$$\prod_{r=1}^{n} (1-\alpha_r)^{-1/2} \begin{vmatrix} 1 & -\rho_{12}\dfrac{\alpha_2}{1-\alpha_2} & \cdots & \rho_{n1}\dfrac{\alpha_n}{1-\alpha_n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ -\dfrac{\rho_{1n}\,\alpha_1}{1-\alpha_1} & -\rho_{n2}\dfrac{\alpha_2}{1-\alpha_2} & \cdots & 1 \end{vmatrix}^{-1/2}$$

If now $-1/2$ is replaced by $-m/2$, there is obtained the *m. g. f.* for the variance in a sample of m. Then setting $-m/2=-p$, $G(\alpha_1, \cdots, \alpha_n)\pi\ (1-\alpha_r)^p$ is expanded by the binomial Theorem and thence an expression for n variate Gamma Type distribution is obtained in terms of Laguerre Polynomials.

The particular case $n=2$, was obtained by W. F. Kibble in his paper "A two variate Gamma Type Distribution," *Sankhya*, vol. 5 (1941), pp. 137–150.

K. S. SINGHWI and P. L. BHATNAGAR, Delhi

Electrostatic Correction and White Dwarfs

An approximate expression for the pressure p of a degenerate electron gas in a pure hydrogen white dwarf derived by taking into account the mutual attraction and repulsion of the proton and the electrons, is

$$p=9{\cdot}913\times 10^{12}\rho^{5/3}\left[1-2{\cdot}161\times 10^{10}\frac{\rho^{1/3}}{T^2}-1{\cdot}040\times 10^{3}\frac{\rho^{-1/6}}{T^{1/2}}\right],$$

where ρ and T denote the density and temperature respectively. The percentage electrostatic correction is calculated for three model white dwarfs. It is found that this correction for Sirius B amounts to 4·5% of the total pressure on the assumption that Sirius B is a hydrogen white dwarf.

A. A. GNANADOSS, Tambaram

The perfect equal temperament and generalized continued fraction

Dewan Bahadur K. S. RAMASWAMI SASTRI, Madras

The evaluation of Indian Mathematics

R. VENKATACHALAM IYER, Trivandrum

Patiganita and the Hindu Abacus

Proceeding on the assumption of the necessity of writing material (a board) for calculation, Datta and Singh derive the meaning of *pāṭīgaṇita* from the word *pāṭī* (a board) which on their own admission does not mean a board in Sanskrit. Internal evidence of the Sanskrit works cited by the authors as dealing with *pāṭīgaṇita* coupled with reference to Sanskrit Lexicons shows the meaning of *pāṭī* to be *paripāṭī* or systematic procedure. There is evidence in the above mentioned Sanskrit works, that numerals were not written down for purposes of calculation.

The mode of calculation prescribed in these works survives by tradition among orthodox Hindu Pandits, who carry out calculations on a flat board with cowries, using no written symbols. The flat board serves the purpose of an Abacus in India.

SYMPOSIUM ON RECENT ADVANCES IN HARMONIC ANALYSIS

The Algebraization of Harmonic Analysis

BY

M. H. STONE [(1)]

§ 1. **Introduction.** Harmonic analysis is that branch of mathematics which deals with the representation of general functions in terms of the exponential functions, or harmonics $e^{i\lambda x}$ as infinite series or integrals :

$$(1)\quad f \sim \sum_{n=-\infty}^{n=+\infty} a_n e^{inx}, \qquad a_n = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(x) e^{-inx} dx$$

(Fourier series)

$$(2)\quad f \sim \sum_{n=1}^{n=\infty} a_n e^{i\lambda_n x}, \qquad a_n = \lim_{T \to +\infty} \frac{1}{2T}\int_{-T}^{+T} f(x) e^{-i\lambda_n x} dx$$

(Bohr-Fourier series)

$$(3)\quad f \sim \int_{-\infty}^{+\infty} a(\lambda) e^{i\lambda x} d\lambda, \qquad a(\lambda) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(x) e^{-i\lambda x} dx$$

(Fourier integral)

$$(4)\quad f \sim \int_{-\infty}^{+\infty} e^{i\lambda x} dA(\lambda), \quad A(\lambda) \text{ composed of monotone functions.}$$

Harmonic analysis is one of the most important parts of classical analysis. It has numerous applications in mathematical physics and in other domains where periodicities, exact or approximate, are significant. In the last two decades or so investigations in the field of harmonic analysis have led to the recognition and exploitation of algebraic aspects of its mathematical structure which had been ignored or neglected in the classical treatments. The basic reason why harmonic analysis has intimate and essential contacts with algebra is the obvious one that the exponential function $e^{i\lambda x}$, by virtue of the functional equation

$$(5)\quad e^{i\lambda(x+x')} = e^{i\lambda x}\, e^{i\lambda x'},$$

is not only periodic but also provides a simple multiplicative representation of the additive group of the real numbers, in terms of which the concept of periodicity is formulated. The purpose of the following remarks is to discuss the historical origin of the modern algebraic trend in the field of harmonic analysis.

§ 2. **Wiener's Tauberian Theorems.** Without underestimating the significance or the influence of contributions touched on later in our discussion, it can fairly be said that **N. Wiener** [2] gave a major impetus to the algebraization of harmonic analysis, through his insistence upon the rôles played by the additive group of the real numbers and the related operation of convolution, defined for Lebesgue-integrable functions by the equation

$$(6) \quad (f * g)(x) = \int_{-\infty}^{+\infty} f(x-y)g(y)dy.$$

Although Wiener had a clear and explicit appreciation of the significance of these algebraic features of harmonic analysis, he continued to use methods of a type familiar in classical analysis. It remained for others to develop the algebraic methods which bring out more fully the essential nature of the theory. Wiener's grasp of the situation comes out very clearly in his application of the methods of harmonic analysis to the treatment of Tauberian theorems. As this application serves also to illustrate in a very striking way the power of the modern algebraic methods, we propose to describe it briefly here.

Wiener starts from the observation that many of the standard Tauberian theorems can be obtained as special instances of an implication of the type

$$(7) \quad \textit{if} \quad \lim_{x \to +\infty} \int_{-\infty}^{+\infty} K_1(x-y)h(y)dy = A(h) \int_{-\infty}^{+\infty} K_1(y)dy,$$

$$\textit{then} \quad \lim_{x \to +\infty} \int_{-\infty}^{+\infty} K_2(x-y)h(y)dy = A(h) \int_{-\infty}^{+\infty} K_2(y)dy.$$

This implication we shall call the *Tauberian implication*; in it K_1 and K_2 are supposed to be Lebesgue-integrable functions while h is bounded and measurable. Wiener determines those conditions on K_1 which will guarantee the truth of the Tauberian implication for *all* K_2 and h. To do so he develops methods based upon the harmonic analysis of the functions occurring in (7) by means of the Fourier integral. Because of the connections between the additive group of the real numbers and the multiplicative group of the complex numbers mediated by the exponential function in accordance with (5), one has the *formal* relations

$$(8) \quad (K * h)^{+}(x) = \int_{-\infty}^{+\infty} e^{-ixy} \left(\int_{-\infty}^{+\infty} K(y-z)h(z)dz \right) dy$$

$$= \int_{-\infty}^{+\infty} e^{ixz}h(z)\left(\int_{-\infty}^{+\infty} e^{-ix(y-z)}K(y-z)dy\right)dz$$

$$= h^+(x)K^+(x) = K^+(x)h^+(x)$$

where $K^+(x) = \int_{-\infty}^{+\infty} e^{-ixy}K(y)dy$ and $h^+(x) = \int_{-\infty}^{+\infty} e^{-ixy}f(y)dy$

are the Fourier transforms of K and h respectively. The relations (8) suggested to Wiener that it might be possible to connect K_1 and K_2 in the Tauberian implication (7) by the simple equivalent relations

$$(9)\quad K_2 = g * K_1 \quad \text{and} \quad (10)\quad K_2^+ = g^+ \cdot K_1^+.$$

On the one hand (9) would lead at once to the verification of the Tauberian implication since one has, when g is Lebesgue-integrable,

$$(11)\quad \int_{-\infty}^{+\infty} K_2(x)dx = \int_{-\infty}^{+\infty}\left(\int_{-\infty}^{+\infty} K_1(x-z)g(z)dz\right)dx$$

$$= \int_{-\infty}^{+\infty} g(z)\left(\int_{-\infty}^{+\infty} K_1(x-z)dx\right)dz$$

$$= \int_{-\infty}^{+\infty} g(z)dz \int_{-\infty}^{+\infty} K_1(x)dx,$$

$$(12)\quad \lim_{x\to+\infty}\int_{-\infty}^{+\infty} K_2(x-y)h(y)dy = \lim_{x\to+\infty}\int_{-\infty}^{+\infty}\left(K_1(x-y-z)\, g(z)h(y)dz\right)dy$$

$$= \int_{-\infty}^{+\infty} g(z)\left(\lim_{x\to+\infty}\int_{-\infty}^{+\infty} K_1(x-y-z)h(y)dy\right)dz$$

$$= \int_{-\infty}^{+\infty} g(z)dz \cdot A(h)\int_{-\infty}^{+\infty} K_1(x)dx$$

$$= A(h)\int_{-\infty}^{+\infty} K_2(x)dx.$$

On the other hand, it is plausible that the factorisation (10) is possible for arbitrary K_2 if and only if $K_1^+(x)$ vanishes for no value of x, $-\infty < x < +\infty$. What Wiener accomplishes, by rather indirect and highly ingenious methods, is to show[3] that the latter condition on K_1 is both necessary and sufficient for the validity of the Tauberian implication (7) for *all* K_2 and h. An interesting lemma employed in his proof states the following remarkable property of Fourier series[4]: if $f(x)$ has an absolutely convergent Fourier series (1) (so that

$$\sum_{n=-\infty}^{n=+\infty} |a_n| < +\infty)$$

and if $f(x) \neq 0$ then $1/f(x)$ also has an absolutely convergent Fourier series.

Wiener made a further important observation[5]—namely that (9) is essentially equivalent to the possibility of approximating to K_2 by a linear combination of the translates $K_1(x-a)$ of K_1 in the sense that the integral

$$\int_{-\infty}^{+\infty} |K_2(x) - \sum_{k=1}^{n} c_k K_1(x-a_k)|\,dx$$

can be made small. He showed that the condition $K_1^+(x) \neq 0$ is necessary and sufficient for the translates of K_1 to generate the entire Lebesgue-space $L_1(-\infty, +\infty)$ in this sense. The deduction of the Tauberian implication is then immediate since

$$\limsup_{x\to+\infty} \left| \int_{-\infty}^{+\infty} K_2(x-y)h(y)dy - A(h) \int_{-\infty}^{+\infty} K_2(y)dy \right|$$

differs but little from

$$\limsup_{x\to+\infty} \left| \int_{-\infty}^{+\infty} \sum_{k=1}^{n} c_k K_1(x-a_k-y)h(y)dy - A(h) \int_{-\infty}^{+\infty} \sum_{k=1}^{n} c_k K_1(y-a_k)dy \right| = 0.$$

§ 3. **Weyl's Theory of Almost Periodic Functions ; the Peter–Weyl Theory of Compact Lie Groups ; the Pontrjagin Duality Theorem.** In a sense, Wiener's work, part of which we have just reported, showed that the convolution operation could be studied by means of harmonic analysis. On the other hand the more or less contemporaneous work of H. Weyl,[6] whose influence upon his own investigations Wiener points out, demonstrated that some of the central problems of harmonic analysis could be studied by means of the convolution operation. Wiener himself approached his theory of the spectrum of an arbitrary function from the latter point of view[7]. In the theory of almost periodic functions, with which both Weyl and Wiener were concerned, the convolution operation is defined as a mean :

$$(13) \quad (f * g)(x) = \lim_{T\to+\infty} \frac{1}{2T} \int_{-T}^{+T} f(x-y)g(y)dy.$$

It depends explicitly upon the group-operation underlying the concepts of periodicity and almost periodicity. The theory of Weyl is based upon a study of the integral equation

(14) $f * g = \alpha g$

and leads to the complete harmonic analysis of an arbitrary almost periodic function f. We may note that when f has the form (2) the solutions of (14) are, at least formally, given by $g = e^{i\lambda_n x}$, $\alpha = a_n$. Subsequently Peter and Weyl[8] applied similar arguments in a study of other groups. If G is a compact Lie group (or, as we now know, any locally compact topological group) it has a left-invariant integral[9] (uniquely determined up to a constant multiplier) in terms of which a convolution-operation upon functions of the group elements can be defined as

(15) $$(f * g)(x) = \int_G f(xy^{-1})g(y)dy.$$

The theory of the integral equation (14) then results in a determination of the irreducible orthogonal or unitary representations of the group G and in the expression of an arbitrary function f as a series of functions occurring in the matrices corresponding to such representations. The theory of Fourier series appears as a special case when the group G is taken to be the additive group of the real numbers modulo 2π. Since it is now known how to identify the almost periodic functions on a general locally compact topological group with the continuous functions on a related compact topological group, the Weyl and Peter-Weyl theories can be considered as unified. Here explicit mention should be made of the investigations of J. Von Neumann[10], A. Weil[11], and I. E. Segal[26].

These theories revealed a vague kind of duality between a group and its representations, embracing as a special case that between a function and the ensemble of its Fourier coefficients. It was Pontrjagin [12] who first gave a complete and satisfactory treatment of this duality in the case of locally compact abelian groups. He showed that each such group G has a dual group G' whose elements are the distinct irreducible representation-matrices (necessarily of order one) for G, the connection between the two groups being reciprocal. If one group is compact the other must be discrete and conversely, a fact which neatly explains the difference between the representations by Fourier series and by Fourier integrals. Pontrjagin's duality theorem opened up for A. Weil[13] and later for others the possibility of constructing a generalized harmonic analysis on an arbitrary locally compact abelian group G and its dual G': if g and g' are elements of the respective groups, $\langle g, g' \rangle$ may denote the complex number obtained by calculating one as a function of the other; and formulas of the Fourier-Plancherel type

$$(16) \quad F(g) = \int_{G'} F'(g')<g,g'>dg', \quad F'(g') = \int_G F(g)\overline{<g,g'>}dg,$$

where the invariant integrals have been suitably normalized, can then be established. The case where G and G' coincide with the additive group of the real numbers yields the relation $<g,g'> = e^{igg'}$ and the standard Fourier-Plancherel formulas. Thus it can be seen that the theory of harmonic analysis has now been firmly attached to the theory of groups and has to this extent been "algebraized". A very interesting outstanding problem of group theory is that of extending the Pontrjagin duality theorem to cover general non-commutative locally compact groups. Tannaka[14] has been able to connect each compact group with a dual object which is an algebra formed from its representations. Cartan, Godement, and Segal have been able to obtain analogues of the Plancherel theorem (16) for the so-called unimodular groups[15].

§ 4. **The Algebra of Bounded Continuous Functions.** It might have been expected that the importance of the equations (10) and (14) in the theories of Wiener and of Weyl would lead to a direct study of the algebra of the convolution-operation, especially as Weyl gave a very clear exposition of the meaning of this algebra for group-theory in his book "Group Theory and Quantum Mechanics". Historically, however, the development of the necessary techniques for such a study seems to have resulted from the attention given to another problem which involves an altogether different and apparently unrelated algebraic aspect of analysis. For reasons which it is unnecessary to discuss here, beyond remarking that the ultimate applications in view were concerned with the theory of operators in Hilbert space, Stone[16] found occasion to investigate the algebraic structure of the ring of all continuous functions on a compact Hausdorff space, determining in particular all its closed ideals in the standard norm-topology, where the norm of f is given by

$$(17) \quad \|f\| = \sup_x |f(x)|.$$

He raised the question as to what properties of an abstract ring would permit its identification with the ring of all continuous functions on some compact space[17]. Various aspects of this question and of the corresponding lattice-theoretic question were the object of considerable attention by mathematicians in the period 1935–1940. Approximately simultaneous solutions were obtained for the ring problem by Gelfand[18] and by Stone[19], and for the lattice problem by Kakutani[20] and by M. and S. Krein[21]. An essential feature of

all these solutions is the use of a method developed by Stone[22] for the representation theory of Boolean rings and later explicitly formulated in much broader terms by McCoy and Montgomery[23]. This method consists in representing the elements of an abstract algebraic ring by functions defined over the set of its maximal ideals. At the same time I. E. Segal[24] was initiating a study of the algebra of the convolution operation for the additive group of the real numbers; both Gelfand[25] and he[26] obtained algebraic proofs of Wiener's principal Tauberian theorems, of which we shall now give a brief account.

Gelfand[27] starts with the consideration of a normed commutative algebra with unit over the complex numbers, requiring that it be complete in its norm-topology. Such a system is now commonly called a (complex) Banach algebra. There is no loss of generality in supposing that the norm is one for which $|xy| \leqq |x|\,|y|$. The maximal ideals in such an algebra are closed and are precisely the kernels of the homomorphisms of the given algebra A into the complex field. To represent A by an algebra of complex-valued functions it suffices to associate with each x in A a function f_x over the set of maximal ideals M where $f_x(M)$ is simply that complex number into which the homomorphism $A \rightarrow A/M$ carries x. The functions f_x are bounded, in accordance with the relation

$$(18) \quad \sup_M |f_x(M)| = \lim_{n\to\infty} |x^n|^{1/n} \leqq |x|.$$

The elements x such that $\lim_{n\to\infty} |x^n|^{1/n} = 0$ constitute an ideal which is identical with the intersection of all the maximal ideals M. This ideal is called the radical of A and its elements are called generalized nilpotents. Evidently the representation of A by the indicated algebra of functions is isomorphic if and only if 0 is the only generalized nilpotent in A; and it is both isomorphic and isometric if and only if $|x^2| = |x|^2$. On topologizing the set of all maximal ideals in the weakest possible manner such that the functions f_x all become continuous, one finds that the resulting space is a compact Hausdorff space. On the basis of the generalized Weierstrass approximation theorem, as originally given by Stone[26], it is then possible to obtain necessary and sufficient conditions for A to be isomorphic and isometric to the algebra of *all* continuous complex functions on this space.

The study of the convolution-operation in $L_1(-\infty, +\infty)$ is easily brought under Gelfand's theory of Banach algebras, since with

$$\|f\| = \int_{-\infty}^{+\infty} |f(x)|dx$$

we obviously have $\|f * g\| \leq \|f\| \|g\|$ by a direct calculation. As there is no unit element one has to be adjoined. This can easily be done without modifying the norm since the sequence

$$g_n = \frac{\sin^2 nx}{\pi n x^2}$$

in L_1 has the property that

$$(19) \quad \|f - f * g_n\| \to 0, \ \|g_n\| = 1,$$

in accordance with a simple argument due originally to Féjer[29]. In fact it is enough to consider all formal expressions $\alpha e + f$ where e is the desired unit element, introducing the definitions

$$(\alpha e + f) + (\beta e + g) = (\alpha + \beta)e + (f + g),$$
$$(20) \quad (\alpha e + f) * (\beta e + g) = \alpha\beta e + \beta f + \alpha g + f^* g,$$
$$\|\alpha e + f\| = \sup_g \|\alpha g + f * g\| / \|g\|.$$

This amounts to treating $\alpha e + f$ as a symbol for that bounded linear operator in L_1 which carries g into $\alpha g + f * g$. The maximal ideals in this Banach algebra A are the kernels of the homomorphisms $\alpha e + f \to \alpha + l(f)$ where l is a bounded linear functional on L_1 with the special property that $l(f * g) = l(f) l(g)$. As is well-known, l necessarily has the form

$$l(f) = \int_{-\infty}^{+\infty} f(x)\phi(x)dx$$

where ϕ is bounded and measurable. It is easily seen that the relation $\phi(x + y) = \phi(x)\phi(y)$ must hold almost everywhere and hence that unless ϕ vanishes almost everywhere it has the form $\phi(x) = e^{-i\lambda x}$ almost everywhere. Consequently l either vanishes identically or has the form

$$l(f) = \int_{-\infty}^{+\infty} f(x) e^{-i\lambda x} dx = f^+(\lambda) \text{ for all } f \text{ in } L_1.$$

Turning now to Wiener's Tauberian theorem, we first observe that the totality of all functions K_2 validating the Tauberian implication (7) for all h is a closed ideal I in A which contains K_1: for if K_2 validates (7) so also does $K_2 * (\alpha e + f) = \alpha K_2 + K_2 * f$ for any f in L_1, as is indicated by (11) and (12); if $K_2^{(n)}$ validates (7) and $\|K_2 - K_2^{(n)}\| \to 0$ then K_2 also validates (7), as is indicated in the special case discussed at the end of § 2; and, when K_2 is taken to

be K_1, the implication (7) is trivially validated. The condition $K_1^+(\lambda) \neq 0$ shows that the only maximal ideal containing I (and in particular its element K_1) is the ideal corresponding to the case where the functional l vanishes identically. As this ideal obviously coincides with L_1, the proof of Wiener's Tauberian theorem thus reduces to showing that $I = L_1$. Since it cannot be established in general that an arbitrary closed ideal in a Banach algebra is the intersection of the maximal ideals containing it, this relation has to be proved by a supplementary argument suited to the particular circumstances. For this purpose we shall follow a proof due to I. E Segal(30). Let K_2 be a function in L_1 such that $K_2^+(\lambda)=0$ outside a fixed bounded closed interval Δ. We shall show that K_2 is in I. Being the intersection of maximal ideals, the totality of elements $\alpha e+f$ such that $\alpha+f^+(\lambda)=0$ for every λ in Δ is a closed ideal D in A. It is easy to find in L_1 a function g such that $g^+(\lambda)=1$ for every λ in Δ. Obviously the element $e-g$ is in D since $1-g^+(\lambda)=0$ for every λ in Δ. The smallest ideal containing both I and D must contain e: for otherwise, the maximal principle (Zorn's principle) would furnish a maximal ideal containing both; and yet the only possible such ideal would be L_1, which clearly does not contain the element $e-g$ in D. Thus e can be expressed as the sum $f+h$ of an element in I and an element h in D; in other words, I contains an element f such that $e-f$ is in D or, equivalently, such that $f^+(\lambda)=1$ for every λ in Δ. We now consider the element $K_2 * f$ in I. The difference $K_2 - K_2 * f$ has the property that its Fourier transform $K_2^+(\lambda)(1-f^+(\lambda))$ vanishes identically: $1-f^+(\lambda)=0$ when λ is in Δ and $K_2^+(\lambda)=0$ when it is not. Hence $K_2 - K_2^+ f$ vanishes almost everywhere(31) and K_2 is therefore in I. If K_2 is an arbitrary element in L_1 it is the limit of the elements $K_2 * g_n$ as indicated in (19). Now direct calculation shows that $g_n^+(\lambda)=1-|\lambda|/2_n$ when $|\lambda| \leq 2n$ and $g_n^+(\lambda)=0$ otherwise. Hence $(K_2 * g_n)^+ = K_2^+ g_n^+$ vanishes outside the interval $\Delta=[-2n, +2n]$ and $K_2 * g_n$ is in I. Since I is closed, K_2 must also be in I. This completes the proof.

By a more elaborate version of the argument just presented, Segal(32) obtained generalizations of the Wiener theorem which are of considerable interest and importance. We should note also that Wiener's lemma on absolutely convergent Fourier series, which we mentioned above, likewise has a simple algebraic proof due to Gelfand(33).

§ 5. **L. Schwartz's Theory of Distributions.** An entirely different algebraic aspect of the theory of Fourier series and Fourier

integrals has been indicated recently by L. Schwartz[34] as an application of his theory of distributions. Considerations of space preclude any adequate discussion of this remarkable theory, but our theme demands explicit mention of it. We shall content ourselves with a brief indication of the manner in which this theory eliminates all convergence difficulties and permits free algebraic reckoning with the Fourier series for distributions and their derivatives. A distribution is a continuous linear functional on the (suitably topologized!) space of all infinitely differentiable functions with compact nucleus[35]. A function f is treated as that distribution T which takes ϕ into

$$\int_{-\infty}^{+\infty} f\phi dx.$$

The derivative of a distribution T is the functional T' which takes ϕ into $-T\phi'$. It turns out that any series

$$\sum_{n=-\infty}^{n=+\infty} a_n e^{inx}, \ a_n = O(n^k),$$

considered as a sum of distributions, converges to a distribution T whose derivative is represented by the series

$$\sum_{n=-\infty}^{n=+\infty} in \ a_n e^{inx}.$$

For if we integrate by parts a sufficient number of times we see that

$$\int_{-\infty}^{+\infty} \sum_{n=-N}^{n=+N} a_n e^{inx}\phi(x)dx = \int_{-\infty}^{+\infty} \sum_{n=-N}^{n=+N} \frac{a_n}{(-in)^l} e^{inx}\phi^{(l)}(x)dx$$

converges as $N \rightarrow +\infty$ to a distribution T defined by

$$T\phi = \int_{-\infty}^{+\infty} f\phi^{(l)}dx, \ f = \sum_{n=-\infty}^{n=+\infty} \frac{a_n}{(-in)^l} e^{inx};$$

and the derivative T' is then given by

$$T'\phi = -T\phi' = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{n=+\infty} a_n \frac{(in)}{(-in)^{l+1}} \phi^{(l+1)}(x)dx$$

$$= \lim_{N\rightarrow+\infty} \int_{-\infty}^{+\infty} \sum_{n=-N}^{n=+N} (in)a_n e^{inx}\phi(x)dx.$$

Thus the type of purely algebraic calculation which the physicist and the engineer have always been eager to employ in the applications of Fourier analysis appears to be fully justified within the realm of distribution theory.

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Footnotes

(1) This paper was delivered as a lecture on December 27, 1949, in connection with the Symposium on Harmonic Analysis organized by the Sixteenth Biennial Conference of the Indian Mathematical Society, held at Madras.

(2) N. Wiener, "*The Fourier Integral and Certain of Its Applications*", Cambridge, England, 1933.

(3) N. Wiener, loc. cit.[2], pp. 72–97, culminating in proofs of Theorems 4, 5, 6, 7 as stated on pp. 73–74.

(4) N. Wiener, loc. cit.[2], pp. 86–91, culminating in Lemma 6_{16}.

(5) N. Wiener, loc. cit.[2], pp. 97–99.

(6) H. Weyl, "*Integralgleichungen und fastperiodische Funktionen*", Mathematische Annalen, 97(1927), pp. 338–356.

(7) N. Wiener, loc. cit.[2], pp. 150–199.

(8) H. Weyl and F. Peter, "*Die Vollständigkeit der primitiven Darstellungen einer geschlossenen continuirlichen Gruppe*", Mathematische Annalen, 97 (1927). pp. 737–755.

(9) The integral with respect to left-invariant Haar measure as discussed, for example, in I. E. Segal, "*Invariant Measures on Locally Compact Spaces*" Journal of the Indian Mathematical Society, 13(1949), pp. 105–130.

(10) J. von Neumann, "*Almost Periodic Functions in a Group*", Transactions of the American Mathematical Society, 36(1934), pp. 445–492.

(11) A. Weil, "*L'intégration dans les groups topologiques et ses applications*", Actualités Scientifiques et Industrielles, 869, Paris, 1940.

(12) L. Pontrjagin, "*The Theory of Topological Commutative Groups*", Annals of Mathematics, 35(1934), pp. 361–388.

(13) A. Weil, loc. cit.[11], Chapter VI.

(14) S. Tannaka, "*Dualität der nicht-kommutativen Gruppen*", Tohoku Journal of Mathematics, 53(1939).

(15) References to work published or announced by H. Cartan, R. Godement, and I. E. Segal are not available to me at the time of writing.

(16) M. H. Stone, "*Applications of the Theory of Boolean Rings to General Topology*" Transactions of the American Mathematical Society, 41(1937), pp. 375–481 especially pp. 455–459, 465, 473.

(17) M. H. Stone, loc. cit.[16], p. 476, first paragraph.

(18) I. Gelfand, "*Normierte Ringe*" Matematicheskii Sbornik, N.S. 9(51)(1941), pp. 3–24; announced in Doklady, Akademia Nauk, U.S.S.R., 23(1939). pp. 430–432.

(19) M. H. Stone, "*A General Theory of Spectra, I*", Proceeding of the National Academy of Sciences, U.S.A., 26(1940), pp. 280–283.

(20) S. Kakutani, "*Concrete representation of Abstract M-Space*", Annals of Mathematics, 42(1941), pp. 994–1024; see also "*Weak topology, bicompact sets, and principle of duality*", Proceedings of the Imperial Academy of Sciences, Japan, 16(1940), pp. 63–67.

(21) M. and S. Krein, "*On an Inner Characteristic of the Set of All Continuous Functions Defined on a Bicompact Hausdorff Space*", Doklady, Akademia Nauk, U.S.S.R. 27(1940), pp. 427–430.

(22) M. H. Stone, "*The Theory of Representations for Boolean Algebras*", Transactions of the American Mathematical Society, 40(1936), pp. 37–311.

(23) N.H. McCoy and Deane Montgomery, "*A Representation of Generalized Boolean Rings*", Duke Mathematical Journal, 3(1937), pp. 455–459.

(24) I. E. Segal, "*The Group Ring of a Locally Compact Group, I*", Proceedings of the National Academy of Sciences, U.S.A., 27(1941), pp. 348–352.

(25) I. Gelfand, "*Uber absolut konvergente trigonometrische Reihen und Integrale*", Matematicheskii Sbornik, N.S. 9(51) (1941), pp. 51–56.

(26) I. E. Segal, "*The Group Algebra of a Locally Compact Group*" Transactions of the American Mathematical Society, 61(1947), pp. 69–105, especially pp. 88–89, 91–93.

(27) I. Gelfand, loc. cit.[18].

(28) M. H. Stone, loc. cit.[16] pp. 466–469; and "*The Generalized Weierstrass Approximation Theorem*", Mathematics Magazine, 21(1948), pp. 167–183, 237–254.

(29) It will be recognized at once that $g_n(x-y)$ is the familiar Féjer kernal occurring in the study of the arithmetic means of the partial sums of a Fourier series.

(30) I. E. Segal, loc. cit.[26].

(31) The well-known theorem that if K is in L_1 and K^+ vanishes identically then K vanishes almost everywhere can easily be proved on the basis of (19); for if $g_n^+(\lambda)$ is computed (with the result indicated a few lines farther along in the text), it is seen that the function $e^{i\lambda(x-y)}g_n^+(\lambda)K(y)$ is Lebesgue-integrable as a function of λ and y, and that calculation of its integral by iterated integration in the possible orders yields the relation $K^*g_n = 0$.

(32) I. E. Segal, loc. cit.[26].

(33) I. Gelfand, loc. cit.[25].

(34) Laurent Schwartz, "*Généralisation de la notion de fonction, de dérivation, de transformation de Fourier, et applications Mathématiqus et Physiques*", Annals de l'Université de Grenoble, 21(1945), pp. 57–74.

(35) The nucleus of a numerical function is the closure of that set where its values are different from zero.

SYMPOSIUM ON FLUID MECHANICS

Introductory Speech by Prof. O. G. Tietjens

When I was invited to deliver a lecture at this conference of the Indian Mathematical Society, I felt rather uneasy because I am neither a "Pure Mathematician" nor even an "Applied Mathematician". I could not think of anything that I could tell you that would be of interest to you from a mathematical point of view.

However, when I heard that also problems of Hydrodynamics were to be discussed at this conference, I changed my somewhat reluctant attitude and gladly accepted the invitation. Later I was asked to preside over a Symposium on "Fluid Mechanics." I do not know why this honour was conferred just on me, but I take it as a sign of courtesy to somebody who has come to this country only a year ago. I appreciate, indeed, this gracious proof of hospitality.

The idea of having a Symposium on "Fluid Mechanics" is a very happy one. I think it is extremely timely to stress the importance of Fluid Mechanics in these days where also in India the problems of aviation, irrigation, wind mills and many others will soon be of the greatest significance.

Modern Fluid Mechanics, as it has been developed during the last thirty years, has not found yet its due recognition at the various Universities in India. It is more the father and mother of Fluid Mechanics that are generally known. By "father" and "mother", I mean "Hydrodynamics" and "Hydraulics". Two names so similar in sound, and yet, the subject matter of both how different!

Hydrodynamics is the sphere for the mathematician where he could apply his methods, but where he unfortunately has to simplify his equations to an extent that the results only too often do not agree at all with experience. Instead of a real fluid he assumes an ideal fluid, i.e. a continuum having no viscosity and being not compressible. But the result of his calculations are that a body moving within such an ideal fluid does not experience any resistance or drag whatsoever, or that the pressure drop of an ideal fluid moving through a pipe is zero. What is the use of such calculations?

In Hydraulics, on the other hand, the problems are attacked in quite a different manner, not by Mathematicians but by Engineers.

Visualising that the mathematical difficulties were apparently too great to be overcome even by Mathematicians and realising that they themselves, after all, did not have the necessary knowledge, these engineers were forced to solve their problems by making experiments. In order to answer the question as to the resistance of a body moving through a fluid, they made experiments and determined certain coefficients of drag. In order to determine the pressure drop of a fluid moving through a pipe they made the test and again found certain coefficients. For all and everything they found "coefficients".

Whereas the classical Hydrodynamics had in many cases lost the connection with reality, the old Hydraulics was disintegrated into a science of coefficients. In the classical Hydrodynamics there was a logical system, and starting from simple physical conceptions like velocity, pressure, etc., one finally came to a solution of the problem, which, however, as already mentioned, quite often was in contradiction with experience. In the old-fashioned Hydraulics there was practically nothing like a principle of order so that it was finally felt, that experiments, although necessary to check results which were theoretically obtained, or helpful for an intuitive way, never were sufficient for a thorough understanding.

It was the German Mathematician Felix Klein who tried to bring together Hydrodynamics and Hydraulics, to bring together theory and practice. And he was very fortunate in discovering, so-to-speak, a young German scientist and engineer whose life later on was to be devoted to this synthesis of Hydrodynamics and Hydraulics. The name of this man is Ludwig Prandtl. Felix Klein anticipated the genius of this man and offered Ludwig Prandtl at an age of about 25 years a Full-Professorship at the University of Gottingen. In this way Prandtl became the father of what we now call "Fluid Mechanics." With the conception and the theory of the boundary layer he laid the corner-stone of his life's work.

In the meantime great achievements have been made in two-and three-dimensional airfoil theory, propeller theory, theory of turbulence, to mention only a few. In recent years also the flow of compressible fluids, i.e. of gases has been considerably developed.

FILM DEMONSTRATION ON BOUNDARY LAYER FLOW

A film and some slides were shown illustrating a two-dimensional flow around bodies of various shapes. The bodies were moved through water and the flow on the water surface made visible by fine aluminium powder floating on the water surface. Besides the flow around bodies with sharp edges like airfoils showing the generation of the starting vortex, the flow around blunt bodies was also shown. In a close up film the behaviour of the flow within the boundary layer could be studied, and it was clearly shown that certain fluid particles within the boundary layer came to a stand-still due to the effect of viscosity and then started to flow backwards in the boundary layer due to the pressure distribution imposed on the boundary layer by the flow outside the boundary layer.

Finally it was shown that the flow through a diffusor of very large opening angle could be influenced by sucking off the boundary layer. Whereas under normal conditions the flow lines separated from the walls of the diffusor because of the very large opening angle, they did adhere to the walls when the boundary layer was sucked off.

SYMPOSIUM ON TIME SERIES

Prof. Herman Wold, in his opening address at the Symposium, dealt in general with the theory of Stochastic processes in relation to problems in time series. He covered a wide range of topics including (*a*) auto-regressive models, (*b*) moving average models, (*c*) correlogram tests of these two models. He discussed the conditions under which it would be beneficial to apply these two models (*a*) and (*b*) to practical problems. The large sample test for moving averages on the lines of the test presented by Quenouille for the auto-regressive scheme derived by him was also given during the course of the lecture.

Dr. N. S. Sastry (Bombay) who followed, said that Prof. Wold had dealt with the problems of stationary time series. There is another class namely the evolutive time series, which offers more difficult problems. If there are well defined or known laws of growth or decay which are ordinarily represented by secular trend, and if their effect can be eliminated by some means, the resulting series will be of the stationary type and it can be tackled by the methods indicated by Prof. Wold. Unfortunately in several cases, there might be no well defined laws and even if there might be, they may not be easily discernible. Usually the trend is estimated either by some algebraic curve or by the method of weighted or unweighted moving averages. These involve certain assumptions which should be clearly borne in mind. Further, the elimination of this trend may bring out certain new changes in the structure of the time series and this also should be recognised in interpreting any series of figures.

In studying the time series of prices of bullion (silver and gold), Mr. Bhat of the Reserve Bank used Spencer's 21-point formula to eliminate the trend and the autoregressive periods for three sequences 1923–31, 1932–35 and 1936–40 to silver were 7·0, 8·9 and 10·5 respectively. When experimental series were constructed from random numbers of 50 and 100 items and the autoregressive periods were calculated for the two different types, after using Spencer's formula for trend elimination, in all cases the autoregressive period came round about 3·0; to be more precise they were 3·1 and 3·1 for the two 50-item series and 2·8, 2·8 for the other two 100-item series. This shows that the use of Spencer's formula is introducing a spurious period on oscillation. The problem is to correct the observed periods for the period superimposed by the Spencer's formulae. A lot of research on different types of trends and the results of elimination has to be made to grasp their full effects.

Dr. C. R. Rao (Calcutta) described how the theory of discriminant functions could be used in deciding whether the observed correlogram is of the autoregressive or moving average type. If k was the number of correlations available for testing the goodness of fit of a hypothetical correlogram, then the decision function lead to a division of the space of the serial correlations into four portions for the possible decisions—Moving average, Probably Moving average, Probably Autoregressive and Autoregressive. If a sequential test was followed so that successive serial correlations were computed only when no decision is reached, a slight change was needed in Wald's sequential ratio test because there was a limit to the number of computed correlations. The Wald limits $(1-\alpha)/\beta$ and $\alpha/(1-\beta)$ were changed to the closer limits $(1-\alpha-\delta_1)/\beta$ and $\alpha/(1-\beta-\delta_2)$ where δ_1 and δ_2 were suitably determined. The method of computing δ_1 and δ_2 was to be explained. The latter limits were useful in sequential test procedures when it was known that not more than a fixed number of observations could be taken. If no decision was reached up to the last observation then the sequential procedure might be truncated in the usual way.

THE ANALYSIS OF CONSUMER'S DEMAND

By Dr. Hermann Wold

(Summary of three lectures by Dr. Hermann Wold, Professor of Statistics, University of Upsala, Sweden; two lectures were delivered under the auspices of the University of Madras and one lecture at the Indian Mathematical Conference, Madras.)

The national income in Sweden has risen steadily during the last century, at the present moment amounting to some 3000 rupees per person per year. In the course of this development the consumers' demand for food has shifted from the cheaper cereals to the more expensive animal foods, the consumers having gradually become more free to follow their preferences in the demand for foodstuffs. At present the consumption is 60% vegetable foods, and 40% animal foods. Now it is reasonable to forecast that there will be a continued rise in the national income, say by 2% a year, and since the Swedish population is practically stable, there will be a corresponding increase in the average income. Under such circumstances the shift from cereals to animal foods will continue. For the rational planning of agricultural production, it is of course very important to estimate the extent of this shift. For instance, how great will be the percentage of animal foods in 1970? This is a typical example of the questions dealt with in the analysis of consumers' demand.

As a second example, we may refer to the demand for alcoholic liquors. The demand for liquors is in Sweden controlled in two ways: by a system of rationing that limits the purchase of distilled liquors to three litres a month, per person, and by a system of sales tax which raises the price so as to restrict the demand. Now in some quarters, it is argued that the liquor consumption can be effectively controlled by making use of price policy only, and that the system of rationing is unnecessary. This argument leads up to questions such as: How much would prices have to be raised if we wish to abolish rationing, and at the same time avoid an increase in consumption.

These examples are perhaps sufficient to give an idea of the great practical importance of careful studies into the demand patterns of the consumers. Needless to say, such studies are of fundamental importance also from a theoretical viewpoint, demand functions and related notions belonging to the basic tools in economic analysis.

For the empirical study of demand patterns, two kinds of statistical material may be employed. One is family budget data. These record the expenditures of a number of households during a week, a month, a year or some other period, the account being specified in detail for various kinds of food, for housing, servants and all other items of the budget. By grouping the families according to their income, these data will show how the demand for each budget item varies with the income. For instance, for a given increase in the family income it is found that the expenditure on food and other necessities will show a rather small increase; in other words, the demand is *inelastic* with respect to income changes. For certain food-stuffs, however, such as butter, eggs, and other animal foods, as well as for commodities of luxury in general, demand is found to be *elastic* with respect to income changes.

The other type of statistics which can be used in demand analysis is market data, i.e. running records of quantities purchased in the market, and on the other hand, records of prices and other market factors which influence the consumers' purchases. Such data may primarily be employed for estimating the elasticity of the demand with respect to changes in price. Again it is found that the demand is inelastic for the basic foods as well as for other necessities, whereas the demand is elastic for luxuries.

Dr. Wold then proceeded to give a summary of the demand elasticities he had found in a detailed study based on Swedish statistics, and undertaken at the request of the Swedish Government.

In the calculation of demand elasticities, the statistical methods involve many tricky points. The whole field has, in the last decade, been the subject of much controversial discussion. Dr. Wold gave an account of the main problems at issue, and he was able to show how fruitful approaches to the various difficult questions could be made by a combined use of economic and statistical arguments.

The demand patterns are of course rather different in India and Sweden. It would be a task of great importance to find out the main lines of the demand structure in India. This field of research is difficult, it is true, but it will quite certainly give rich results to serious students who combine an insight into economic and statistical theory with a genuine interest in the factual habits of the consumers.

SYMPOSIUM ON THE TEACHING OF MATHEMATICS

Chairman: Dr. A. Narasinga Rao, President of the Society

Prof. Marshall H. Stone of Chicago who opened the symposium said that in a changing world the teaching of mathematics had also to change with the times. Things out of date had to be dropped out, new material added from time to time and the methods of teaching modified. Analytical Geometry and Calculus are rightly receiving more and more attention than in the past. From analytical geometry one could introduce trigonometric functions by means of the coordinates of points connected with a segment of a circle of angle θ. The addition theorem for angles, the central result in trigonometry, came out very easily by this method. Vectors were an essential part of modern teaching. There was less emphasis on numerical examples and more on the physical significance of the entities considered. The laborious algebraic solution of the cubic and the biquadratic had been rendered obsolete by the calculating machines of to-day. In the present day teaching of mathematics the importance of modern algebra was recognized. Statistics also must have its proper place in the mathematical syllabus. Finally there was a thorough revision of the teaching of Geometry in which the concepts of topology found their proper place in the teaching.

Prof. Tietjens (Bangalore) who followed gave an account of the teaching of Mathematics in German Universities. He said that besides class room lectures, there were frequent occasions for discussions in which students and teachers took part. He said that while there was a stipulated number of lectures, students had more time to read for themselves.

Dr. T. Vijayaraghavan (Madras) referred to the Committee on the Teaching of Mathematics which was appointed at the time of the Waltair Conference and the difficulties which stood in the way of its carrying out its work. He felt that by including a discussion on the teaching of mathematics as part of the biennial conferences, it did not receive sufficient attention and not much progress was made. A separate gathering specially arranged for this purpose might result in more expeditious work.

Sri E. Vinayaka Rao (Madras) said that syllabuses for different stages should be prepared by a meeting of the teachers of mathematics as was done in Engladn.

Dr. Ram Behari (Delhi) raised the question as to what subjects should be taught at the various stages of mathematical education, and at what stage rigour should be introduced. He wanted Prof. Stone to give his views.

Prof. Stone said there should be wide choice in the matter. The courses followed in Chicago, for instance might not be considered suitable here. Some students, he said, needed group theory. Regarding rigour, he was against too early an introduction of it which, he said, created difficulties for the students.

Dr. R. Vaidyanathaswamy (Madras) said that there was a tendency to believe that mathematics should be taught as a formal study in which the teaching should be designed so as to reach certain main results quickly. Such an attitude ignored the fact that the learner's mental mechanism had its own natural lines of activity and that it did many things which apparently seemed to be purposeless, like the child kicking its legs to no purpose. It may be true for instance that the results obtained by the methods of geometrical conics could be obtained more easily by the methods of analytical geometry. All the same the methods of geometrical conics made a powerful appeal and was a natural activity at a certain stage of mathematical development. It was necessary to have a better understanding of the workings of the human mind before one condemned the inclusion in the course of topics which had an important place in the history of mathematics.

In the course of his concluding remarks the Chairman said that this question of reform of the teaching of mathematics was an important and urgent one and hoped that it would be possible to arrange for a special gathering of the Society to discuss this problem fully.

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GENERAL BODY MEETING OF THE SOCIETY

Wednesday the 28th December 1949

Chairman: DR. A. NARASINGA RAO, President of the Society

1. A condolence resolution regarding the death of two members of the Society—Mr. Abdulla Butt and Rev. Father Pruvot was moved from the Chair and passed, all the members standing.

2. The Chairman then moved a resolution which ran as follows:

" This conference of the Indian Mathematical Society considers the introduction of decimal coinage and of the metric system of weights and measures in this country highly desirable."

Pointing out the advantages of decimal coinage and the metric system, the Chairman explained that it made calculation easier so that even those without good education could not be easily deceived, and that a good deal of time now spent in the mastery of various conversion tables by pupils could be saved and used to raise the standard of mathematical education.

The resolution was passed unanimously after a discussion in which Prof. B. N. Prasad (Patna) and Prof. Gunjikar (Bombay) took part.

3. The Chairman then elicited the views of the members of the Society on certain changes in the constitution which were under consideration. These related to (i) the change of the name "managing committee" of the Society into the "Council" of the Society (ii) the editors of the *Journal* and the *Student* to be ex-officio members of the Council (iii) the procedure relating to the biennial election of the office bearers. It was pointed out by Dr. Ram Behari that some of the suggestions regarding elections were already embodied in the existing rules and required only a strict application. The Chairman said that the views expressed at the meeting would be kept in mind in submitting a draft revised constitution for the approval of the members. Prof. B. N. Prasad, Dr. K. Chandrasekharan, Prof. S. Raghavachary, Dr. Ram Behari, Dr. S. Minakshisundaram, Dr. C. N. Srinivasiengar and the Chairman took part in the discussion.

4. The Chairman then placed before the General Body an offer from Dr. R. M. Alagappa Chettiar to house the library of the Indian Mathematical Society at the newly founded Ramanujan Institute, Karaikudi and to provide free of cost the services of a librarian and

to meet the expenses of transport of the books and periodicals from Poona, its present location, to Karaikudi. The Chairman explained that the Fergusson College had kindly accommodated the library ever since the foundation of the Society and the Society was grateful for the help. The library had, in the meanwhile grown, and there was acute shortage of space. The committee of the Society had considered the proposal and was recommending it to the General Body for acceptance. The only condition attaching to the offer was that the Library should remain in its new home for at least two years. After a discussion in which Prof. B. N. Prasad, Prof. Gunjikar, Prof. Chandratreya, Prof. D. D. Kapadia and Prof. T. Vijayaraghavan took part, the General Body passed three resolutions, one approving of the transfer of the library to Karaikudi, another thanking Dr. Alagappa Chettiar for his kind offer and the third thanking the authorities of the Fergusson College for their service to the Society in housing its library all these years.

5. There were also various suggestions discussed as to how the library might function more efficiently and about the circulation of current periodicals.

FILM SHOW ON ATOMIC PHYSICS

On Wednesday the 28th at 7 P.M. there was a very interesting and instructive film show in the Presidency College Hall. An Educational film entitled Atomic Physics, consisting of 5 reels got down by the British Council and with very considerable trouble cleared from the Customs just that very afternoon by the efforts of Mr. C. R. Hiscocks and Mr. Hodgson, was screened before a large and appreciative audience consisting of Delegates of the Mathematical conference and leading citizens of Madras. The films traced the evolution of our knowledge of the atom from the electron and the positron and neutron, showed the structure of the elements and their relation to the atomic number and positions in the periodic table, indicated the possibility of atomic energy being released from the splitting of the Uranium atom. The use of atomic energy in the making of the atom bomb during the last war and the possibility of putting atomic energy to useful ends during peace time were very clearly indicated.

At the end of the show which was very much appreciated by the audience, Professor G. A. Srinivasan, Secretary of the Reception Committee thanked Mr. Hiscocks and Mr. Hodgson of the British Council for the pains they had taken to show the picture that evening.

EXCURSIONS

The Reception Committee of the Mathematical Conference had arranged for the Delegates two alternative full day excursions. One party of twenty started at 8 A.M. for Tirukalikunram; at 10 getting up the hill they offered worship at the temple and witnessed the sacred kites taking oblation. Then coming down, the party took lunch and left for Mahabalipuram where they were shown round the various places of historical interest. After a pleasant evening spent there, the party returned to the city at 7 P.M.

A second party of about 70 persons made up of delegates, a few reception committee members and volunteers started at 8 A.M. from the Senate House, went first to the Red Hills Lake where Mr. Muniswamy Achari, the Water Works Engineer, of the Corporation and Mr. Krishnan the Assistant Engineer received the party and showed them round the head works at Red Hills and escorted them to Kilpauk Pumping Station and explained to them the details with respect to the filtering, chlorinating, and the distribution of the Madras Water Supply. After the guests had partaken of biscuits and cool drinks kindly provided for them, they drove to the Madras Christian College, Tambaram.

Dr. W. F. Kibble and Mrs. Kibble received the party at the Saint Thomas Hall, took them round the grounds to the play fields and brought them to the Bishop Heber Hall where the guests were treated to a sumptuous lunch. Then they looked round the Arts and Science Blocks and the various Laboratories.

Leaving Tambaram at 1-30 P.M. the party visited the Madras Institute of Technology at Chromepet. Mr. M. Subbaroya Iyer, Mr. M. K. Ranganathan the Director and the members of the staff of the Institute received the party and took them round the electronics and automobile engineering laboratories in course of construction, the Physics and Chemistry Blocks, the workshop and the Hostel Block where the delegates were provided with excellent coffee and refreshments. The vtsitors were well impressed with the great strides the young institution had made during these five months. The specimens of drawing and designs made by the students who had hardly put in

a half year at the institution were an index of the excellence and thoroughness of the training available at the institution.

The next stop was for a couple of hours at the Meenambakkam Observatory and proved very instructive and illuminating. Dr. Sen the Meteorologist and his staff explained to the party the various sections of the observatory. The Sprengthnether Microseism Recorder in action for locating storm centre, the Fan Type Radio Meteorograph to get the upper air temperature, pressure and humidity, distant reading wind instruments and routine observatory instruments such as Hair Hygrograph, Thermograph, Self Recording Main Gauge, Teleprinter system for quick transmission and reception of messages, Preparation of weather charts, weather analysis for forecasting, were clearly explained. The party were given the opportunity of witnessing a Radiosonde balloon released, carrying a transmitting set and the recording obtained on the receiver. Mr. P. R. Krishna Rao, the Regional Director of Meteorology, Madras Circle, had kindly sent down one of his Assistants for explaining the various details.

Leaving Meenambakkam at 4-30 P.M., the party reached Gemini Studios where under kind instructions of Mr. S. S. Vasan and Mr. K. V. V. Sarma, the visitors were shown round the property room, the make up room, the designing room, the studios, and the audition hall. The party had the pleasure of watching a scene from the Telugu version of *Nishan* being shot. The visit ended pleasantly at 6-30 P.M. after the delegates were given a cup of tea and shown a few interesting shorts in the projection room.

VISITS TO THE OFFICES OF THE HINDU

By the kind courtesy of Mr. K. Srinivasan, Managing Editor of the Hindu the delegates of the Mathematical Conference were taken round the various departments of the Offices of the Hindu in two batches on the evenings of the 28th and 29th instant. They were shown the details of the processes commencing from the teleprinter and the lineo composing and terminating in the final printed papers coming down into the packing hall ready cut and folded.

THE SIXTEENTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

LIST OF PERSONS WHO ATTENDED THE SESSION

1.	Abdur Rahman, P. V. ...	Coimbatore.
2.	Balasingasatyanadar ...	Madras.
3.	Bhagwat, G. V. ...	Bombay.
4.	Bhatnagar, P. L. ...	Delhi.
5.	Chandrasekaran, K. ...	Bombay.
6.	Chandratreya, G. L. ...	Poona.
7.	Chandratreya, M. L. ...	Poona.
8.	Chatterjee, T. P. ...	Delhi.
9.	Das, B. C. ...	Cuttack.
10.	Doraiswami, V. K. ...	Mysore.
11.	Ganapathi, S. ...	Tinnevelly.
12.	Ganapathi Iyer, V. ...	Annamalainagar.
13.	Garde, M. M. ...	Poona.
14.	Ghatage, D. ...	Bangalore.
15.	Gnanadass, A. A. ..	Tambaram.
16.	Gunjikar, K. R. ...	Bombay.
17.	Gupta, H. ...	Hoshiarpur.
18.	Iyengar, K. V. ...	Hyderabad.
19.	Kamalamma, K. N. ...	Bangalore.
20.	Kapadia, D. D. ...	Bombay.
21.	Kaprekar, D. R. ...	Devlali.
22.	Kerker, D. W. ..	Poona.
23.	Kibble, W. F. ...	Tambaram.
24.	Krishnamurthi, A. S. ...	Tambaram.
25.	Krishnamurthi, R. ...	Hyderabad.
26.	Krishnaswami Iyengar, A. A. ...	Waltair.
27.	Leelavathi Madhavarao	Bangalore.
28.	Lahiri, D. B. ...	Calcutta.
29.	Lakshmanamurthi, M. ...	Waltair.
30.	Madhava Rao, B. S. ...	Bangalore.
31.	Madhava Rao, C.B. ...	Bangalore.
32.	Mahadevan, S. ...	Madras.
33.	Manikarnikamma, S. N. ...	Bangalore.
34.	Meenakshisundaram, S. ...	Waltair.
35.	Narasinga Rao, A. ...	Waltair.
36.	Natarajan, T. ...	Madura.
37.	Pankajam, Miss S. ...	Madras.
38.	Parameswara Iyer, M. ...	Usilampatti
39.	Pillai, S. S. ...	Calcutta.
40.	Prasad, B. N. ...	Patna.
41.	Racine, Rev. Father. ...	Madras.
42.	Raghavachari, S. ...	Kumbakonam.
43.	Rao, C. R. ...	Calcutta.
44.	Raman, Sir C. V. ...	Bangalore.
45.	Ramanujam, R. ...	Trichinopoly.
46.	Ramaswami, V. ...	Waltair.
47.	Ram Behari ...	Delhi.

48. Rama Rao, N. ... Tirupathi.
49. Ranganathan, V. ... Trichinopoly.
50. Rangaswami Ayyar, K. ... Annamalainagar.
51. Sankaranarayanan, A. ... Virudhunagar.
52. Sastry, N. S. ... Bombay.
53. Sen, D. K. ... Kolhapur.
54. Shah, S. M. ... Aligarh.
55. Sharma, H. G. S. ... Belgaum.
56. Siddiqi, M. R. ... Hyderabad.
57. Singh, A. N. ... Lucknow.
58. Sivaramasastri, S. ... Mysore.
59. Srinivasa Iyengar, C. N. ... Bangalore.
60. Srinivasan, D. K. ... Pudukottah.
61. Srinivasan, M S. ... Tanjore.
62. Srinivasan, S. ... Trichinopoly.
63. Srirangamma. ... Bangalore.
64. Stone, M. H. ... Chicago.
65. Subbarama Iyer, C. V. ... Trivandrum.
66. Suryanarayana Iyer, S. ... Trichinopoly.
67. Thiruvenkatachar, V. R. ... Bangalore.
68. Tietjens, O. G. ... Bangalore.
69 Totadri Iyengar, T. ... Trichinopoly.
70. R. Vaidyanathaswamy. ... Madras.
71. Varma, K. L. ... Kotah.
72. Venkatachala Iyengar, K. ... Bangalore.
73. Venkatachalam Iyer, R. ... Trivandrum.
74. Venkatarama Iyer, M. ... Tumkur.
75. Venkataraman, C. S. ... Trichur.
76. Venkataraman, T. S. ... Alwaye.
77. Venkobarao, N. ... Bangalore.
78. Venkatarayudu, T. ... Waltair.
79. Vijayaraghavan, T. ... Waltair.
80. Wold, Hermann. ... Upsala.

ANNOUNCEMENTS AND NEWS

Mr. N. H. Phadke, M.A., Prof. of Mathematics, Prajapati Bhuvan, Gokhale Road, Dadar, Bombay 28, has been admitted as a member of the Indian Mathematical Society.

Dr. K. Chandrasekharan of the Tata Institute of Fundamental Research, Bombay has been appointed Editor of *The Journal of the Indian Mathematical Society* and Dr. C. N. Srinivasiengar of the Central College, Bangalore as Editor of *The Mathematics Student*.

Sri V. S. Krishnan of Madras has proceeded to Paris for pursuing mathematical studies.

The periodical *Zentralblatt für Mathematik* which publishes short abstracts of Mathematical papers and which had suspended publication during the war has resumed publication.

Dr. R. S. Varma has been appointed Senior Scientist in the Defence Science Organization, Ministry of Defence of the Government of India.

REVIEWS

The Geometry of the zeros of a Polynomial in a Complex Variable

by Morris Marden, Professor of Mathematics, University of Wisconsin, Milwaukee U.S.A. (Published by the American Mathematical Society [531 West 116th, Street, New York]—Mathematical Surveys, Number III) Price $ 5-00181, pages.

The problem of extending to the complex zeros of polynomials the classical Rolle's and Sturm's theorems has attracted a large number of mathematicians beginning with Gauss and Cauchy. The book under review contains an up-to-date self-contained account of all the important results obtained in connection with the above problem. The author of the book is himself a research worker in this field and has made substantial contributions to the subject.

The following is a brief chapter—wise summary of the contents of the book. Chapter I gives some preliminary results like Rouche's and Hurwitz' theorems for use in later chapters. Chapter II deals with generalisations and modifications of the main result, due to Lucas, that any convex region in the complex plane containing all the zeros of a polynomial contains also all the zeros of its derivative and therefore of all its derivatives. By introducing the notion of polar derivatives, the above results are formulated, in Chapter III, in a form invariant for bi-linear transformations of the complex plane. Chapter IV deals with the problem of relative location of the zeros of pairs of related polynomials like apolar polynomials while, in Chapter V, the zeros of the derivative of a rational function is located in relation to its zeros and poles. Chapter VI treats the problem of the relative situation of the zeros of the derivative of a polynomial with respect to specified of zeros of the polynomial itself.

Chapters VII–X study the zeros of a polynomial as functions of its coefficients leading to the analogues of Descarte's rule of signs and Sturm's algorithm for the real zeros of a real polynomial. Chapter VII gives bounds for a specified number p of zeros of a polynomial in terms of all the coefficients while Chapter VIII gives bounds for p zeros in terms of $p+1$ coefficients. Chapter IX studies the analogue of Sturm's theorem by setting up an algorithm for the number of complex zeros of a polynomial in a half-plane or sector while the tenth and last chapter deals with the same problem for circular regions.

There is an extensive bibliography at the end. The research workers in this field will certainly feel thankful to the author for giving in a compact form results on the subject found scattered in various journals. As the author himself remarks, while indicating the present state of development of the subject, the book will prevent unnecessary duplication of known results. The printing and get up of the book are attractive.

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Algebra (Chairman, A. A. Albert): 1, Groups and universal algebra; 2 Structure theory of rings and algebras; 3, Arithmetic algebra; 4, Algebraic geometry.

Analysis (Chairman, Marston Morse): 1, Algebraic tendencies in analysis; 2, Analysis and geometry in the large; 3, Extremal methods and geometric theory of functions of a complex variable.

Applied Mathematics (Chairman, John von Neumann): 1, Partial differential equations; 2, Statistical mechanics; 3, Random processes in physics and communication.

Topology (Chairman, Hassler Whitney): 1, Homology and homotopy theory; 2, Fibre bundles and obstructions; 3, Differentiable manifolds; 4, Topological groups.

The Sections for the presentation of short contributed papers will be as follows: I, Algebra and Theory of Numbers; II, Analysis; III, Geometry and Topology; IV, Probability and Statistics, Actuarial Science, Economics; V, Mathematical Physics and Applied Mathematics; VI, Logic and Philosophy; VII, History and Education. Each member of the Congress may present only one contributed paper and the time allotted for each paper will be ten minutes. Abstracts for such papers should not exceed 400 words in length and must be submitted on blanks which may be secured from the Secretary of the Congress. Abstracts must be in the hands of the Organizing Committee not later than May 15, 1950. The Organizing Committee has decided that contributed papers must be presented in person.

The official languages of the Congress are English, French, German Italian, and Russian.

Entertainment

There will be many interesting entertainment features, including a reception, a symphony concert and a banquet. There will probably be a number of automobile excursions. It is hoped that those visitors who arrive in New York City will be given the opportunity to see the city under the guidance of some mathematician.

Membership in the Congress

Membership in the Congress will be open to all qualified persons, whether they are able to be present in person or not. For regular members of the Congress the fee is $ 15.00; these persons will receive the Proceedings of the Congress and be entitled to participate in the scientific and social features of the Congress. Members of families, accompanying Congress members and not participating in the scientific meetings, may become associate members, for whom the fee is $7 50; they will not present papers or receive the Proceedings, but will be entitled to many of the other privileges of membership. Only members and associate members of the Congress will have the privilege of residing in the Harvard dormitories.

Financial Support

Besides the support from Harvard University, generous subventions have been subscribed for the Congress by Bell Telephone Laboratories, Carnegie Corporation, General Electric Company, Institute for Advanced Study, Massachusetts Institute of Technology, National Research Council, Rockefeller Foundation, Standard Oil Development Company, and several private donors.

Information

Persons who are considering attending the Congress are requested to fill out and return the enclosed card as soon as possible.

Every effort will be made to facilitate the travel of foreign participants to and from the Congress while they are in the United States. The Cooperation Committee will attempt to arrange financial assistance for those visitors to the United States who require it.

Communications should be addressed to:

INTERNATIONAL CONGRESS OF MATHEMATICIANS
Low Memorial Library
531 West 116th Street
New York City 27, U.S.A.

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All communications intended for publication should be sent to the Editor C. N. SRINIVASIENGAR, Central College, Bangalore, South India. Communications relating to the receipt of the Journal, changes of address and the purchase of current or back numbers of this journal should be sent to Prof. S. MAHADEVAN, 2, Bhaskarapuram, Mylapore, Madras 4.

All communications regarding advertisements must be addressed to the Editor.

Remittances of membership fees and of subscriptions should be sent to the Treasurer, Dr. RAM BEHARI, Jubilee Hall, Delhi University, Delhi.